Balanced flow[1]

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The concept of "balanced" flow is the counterpart, in atmosphere–ocean dynamics, of the well known concept of "nearly incompressible" or "effectively incompressible" flow in classical aerodynamics. In aerodynamics, a key aspect of such flow — long recognized as central to understanding the behavior of subsonic aircraft — is that all the significant dynamical information is contained in the three-dimensional vorticity field. This means that the flow has, in effect, fewer degrees of freedom than a fully general flow. More precisely, it means that freely-propagating sound waves contribute only negligibly to the motion.

In atmosphere–ocean dynamics there is a corresponding statement with "vorticity" replaced by "potential vorticity", understood in a suitably general sense. The statement applies to a vast set of cases of rotating, stably stratified fluid flow, for parameter values typical of the atmosphere and oceans. It provides an important key to understanding many of these cases. If the flow can be considered balanced, then all the significant dynamical information is contained in the potential-vorticity field, in the generalized sense. One may "invert" the potential-vorticity field at each instant to obtain the mass and velocity fields. For a more precise statement see the article on "potential vorticity". Again this means that the flow has, in effect, fewer degrees of freedom than a fully general flow. More precisely, balance and invertibility mean that not only sound waves but also freely-propagating inertia–gravity waves contribute only negligibly to the motion. Thus balanced flows can be much simpler to understand than fully general flows.

Cases of fluid flow describable as balanced come under headings such as "Rossby waves", "Rossby-wave breaking", "vortex dynamics", "vortical modes", "vortical flow", "vortex coherence", "blocking", "eddy-transport barriers", "cyclogenesis", "baroclinic and barotropic instability" and other shear instabilities, all of which are related to the fundamental Rossby-wave restoring mechanism or "quasi-elasticity" that exists whenever there are isentropic gradients of potential vorticity in the interior of the flow domain, or gradients of potential temperature at an upper or lower boundary. The concept of balanced flow also enters into the

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theory of wave–mean interaction, in which the mean flow is often considered to be balanced, regardless of the wave types involved. The theory of wave–mean interaction is fundamental in turn to understanding the "gyroscopic pumping" that drives global-scale stratospheric circulations and chemical transports. Indeed, the concept of balanced flow enters, implicitly or explicitly, into almost any discussion of meteorologically interesting fluid phenomena, all the way from regional pollutant transport to planetary-scale teleconnections mediated by Rossby-wave propagation.

Balanced flow has analogs in simple mechanical systems such as the "springy pendulum" composed of a massive bob suspended from a pivot by a stiff elastic spring. Such a pendulum has slow, swinging modes of oscillation in which the relatively fast, compressional modes of the bob and spring are hardly excited: they contribute negligibly to the motion if the spring is stiff enough. The slow, swinging modes correspond to balanced flow, and the fast, compressional modes to sound and inertia–gravity waves. One may describe the swinging modes to a crude first approximation by setting the length of the spring equal to a constant — a "rigid-pendulum approximation". There is a hierarchy of more accurate approximations that allow the spring to change its length in a quasi-static way. In a finite-amplitude, two-dimensional swinging oscillation, the spring is longest when the bob is lowest and shortest when the bob is highest. Such approximations and their ultimate limitations can be studied mathematically via techniques ranging all the way from two-timing formalisms (method of multiple scales) to bounded-derivative theory and KAM (Kolmogorov–Arnol'd–Moser) theory and other dynamical-systems techniques; there is an enormous literature.

A quasi-static description may approximate the pendulum motion with remarkable accuracy; the error may become exponentially small, or even zero in some cases, as the fast–slow timescale separation increases. The key point, though, is that in the quasi-static description the length of the spring evolves as if it were functionally related to the elevation of the bob. This can be exploited to simplify both the mathematical description of the motion and our conceptual understanding of it. The functional relation holds at each instant t, i.e., it holds diagnostically. More precisely, no derivatives or integrals with respect to t are involved, and values of t do not explicitly enter into the definition of the functional relation. The property of being diagnostic, in this sense, is a crucial part of the mathematical and conceptual simplification.

In atmosphere–ocean dynamics the defining property of balance is that an analogous functional relation holds — diagnostic in precisely the same sense. A flow is said to be balanced if the three-dimensional velocity field $\mathbf{u}(\mathbf{x},t)$ is functionally related to the mass field or mass configuration, i.e., to the spatial distribution of mass throughout the fluid system, presumed to be hydrostatically related to the pressure field. (Knowledge of the mass field then implies knowledge of the temperature and potential temperature fields, hence quantities such as, for instance, the available potential energy and the mass under each isentropic surface.)

Such a functional relation between the velocity and mass fields is called a "balance condition" or "balance relation". It provides just enough information to make the potential-vorticity field invertible. The property of being diagnostic means that if one knows the mass field at some instant t, but knows nothing about its time dependence, nor the value of t itself, then the balance relation must nevertheless allow one to deduce the complete three-dimensional velocity field $\mathbf{u}$. It must allow the velocity field to be deduced from the mass field and from the mass field alone.

To the extent that a balance relation holds, it excludes sound waves and inertia–gravity waves from the repertoire of possible fluid motions. The system then has too few degrees of freedom to describe such waves. This generalizes the familiar statement in aerodynamics that an incompressibility condition excludes sound waves. The reduction in degrees of freedom is sometimes expressed by saying that some degrees of freedom are "slaved" to others, or that the evolving states of the dynamical system confine themselves to a "slow manifold" in phase space, having lower dimensionality than the full phase space in which it is embedded. One might say for instance that the velocity field is "slaved" to the mass field. A more careful statement would be that in balanced flows the mass and velocity fields evolve as if they were slaved to each other, to some useful approximation at least. This is like saying that the two-dimensional swinging motion of the pendulum evolves as if the length of the spring and the elevation of the bob were slaved to each other, to some approximation, even though there is no actual mechanical linkage between the two variables.

A standard example of a balance condition or balance relation is the so-called geostrophic relation, which is simple to write and, for typical extratropical parameter values, qualitatively useful though quantitatively not very accurate:

$$\mathbf{u}(\mathbf{x},t) \;=\; \frac{1}{f}\left(-\,\frac{\partial\Phi(\mathbf{x},t)}{\partial y}\,,\;\frac{\partial\Phi(\mathbf{x},t)}{\partial x}\,,\,0\right)\;. \tag{1}$$

Here f is the Coriolis parameter, $\Phi(\mathbf{x},t)$ is the geopotential height (approximately geometric altitude times gravitational acceleration), and three-dimensional position $\mathbf{x}$ is specified using pressure altitude. Thus the horizontal spatial derivatives

$\partial/\partial x$ and $\partial/\partial y$ are taken at constant pressure altitude rather than at constant geometric altitude. This qualifies as a balance relation because of the presumption that the hydrostatic relation also holds, as normally assumed when using pressure as the vertical coordinate. Knowing Φ on each constant-pressure (isobaric) surface is then equivalent to knowing the mass field. So (1) is, as required, a diagnostic functional relation between the velocity field and the mass field. The vertical derivative of (1) is the so-called "thermal wind equation".

The horizontal coordinates x, y are local Cartesian coordinates in a tangent-plane representation. If we also take $f =$ constant, giving us the so-called "f-plane approximation", then (1) asserts not only that $\mathbf{u}$ is slaved to the mass field but also that it is two-dimensionally incompressible or nondivergent, with streamfunction $\Psi = \Phi/f$, so that

$$\mathbf{u}(\mathbf{x}, t) \;=\; \left(-\frac{\partial \Psi}{\partial y}\,,\; \frac{\partial \Psi}{\partial x}\,,\, 0 \right) \;. \tag{2}$$

The geostrophic relation (1) — or relations, plural, if one prefers to think in components rather than vectors — can be motivated as an approximation to the horizontal momentum equation. The validity of that approximation depends on smallness of the Rossby number, or, more precisely, on being able to neglect relative particle (Lagrangian) accelerations against Coriolis accelerations, equivalently relative particle accelerations against f times the right-hand side of (1). The Rossby number, measuring the advective contribution to the relative particle acceleration against the Coriolis acceleration, is usually of the same order as f^{-1} times a typical magnitude of the relative vertical vorticity $\partial v/\partial x - \partial u/\partial y$, $= \nabla^2 \Psi$ if (2) holds. Here u and v are the horizontal velocity components corresponding to x and y, and ∇^2 is the horizontal Laplacian.

The geostrophic relation (1) was historically of great importance in early attempts to understand the dynamics of synoptic-scale weather systems. The history is sometimes discussed under headings such as "Buys Ballot's law", "cyclonic development theory", and "quasi-geostrophic evolution". Buys Ballot's law is a surface observer's counterpart of (1) and was discovered empirically through early work with weather maps.

The modern concept of balance recognizes that, like the rigid-pendulum approximation, (1) is merely the lowest in a hierarchy of more accurate balance relations. The next member is the relation studied by B. Bolin and J. G. Charney in the 1950s, in connection with efforts to develop practical numerical weather prediction. The Bolin–Charney balance relation retains (2) even if f varies with latitude, and redefines Ψ to satisfy

$$\nabla\cdot(f\,\nabla\Psi) \;=\; \nabla^2\Phi+\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) \tag{3}$$

where ∇ is horizontally two-dimensional. Equation (3) is an approximation to the divergence equation, the latter being the result of taking the horizontal divergence of the horizontal momentum equation. The relative particle acceleration is now retained. Its advective part gives rise to the last term of (3), while the remaining, $\partial/\partial t$ part is annihilated when the divergence is taken, because of (2). It is only because there are no $\partial/\partial t$ terms that the relation (3), with (2), qualifies as a balance relation.

Again because of (2), the right-hand side of (3) can be rewritten in terms of a Jacobian in u and v, as $\nabla^2\Phi-2\,\partial(u,v)/\partial(x,y)$, or equivalently a Hessian in Ψ so that

$$\nabla\cdot(f\,\nabla\Psi) \;=\; \nabla^2\Phi \;-\; 2\left\{\frac{\partial^2\Psi}{\partial x^2}\frac{\partial^2\Psi}{\partial y^2} \;-\; \left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2\right\}\,. \tag{4}$$

Regarded as an equation for Ψ when the mass field Φ is given, (4) is not trivial to solve, because of the nonlinear terms on the right. Iterative methods need to be used. The problem of finding Ψ may even become ill-posed for certain mass fields Φ, adumbrating, for one thing, the fact that there exist mass fields that are not even approximately balanceable by any velocity field. A simple thought experiment to make this last point clear would be to pile up the whole of the Earth's atmosphere into a narrow cylinder above the North Pole, leaving a vacuum elsewhere. It is obvious that no velocity field $\mathbf{u}$ can be in balance with such a mass field. Regardless of the choice of $\mathbf{u}$, the free evolution at subsequent times, in any such thought experiment, would involve sound and inertia–gravity waves of enormous amplitude. That is, it would involve gross imbalance.

Balance relations are useful in practice only because naturally-occurring mass fields, or at least smoothed versions of them are, by contrast, often balanceable to good approximation, as Buys Ballot's law reminds us. In most such cases, (4) with suitable boundary conditions is a well-posed nonlinear elliptic boundary-value problem in the flow domain, the primary exception being flows near the equator, where Rossby numbers are not small and (4) may fail to be elliptic, as can be verified from the theory of Monge–Ampère equations. Again the failure of ellipticity adumbrates a physical reality (though not in a way that is quantitatively precise), namely the fact that balance is liable to break down through "inertial" and "symmetric" instabilities near the equator, where f changes sign.

Balance relations still more accurate than (4) can be defined if one is prepared to deal with more complicated sets of equations. The next relation in the hierarchy — to be referred to here as the "generalized Bolin–Charney balance relation" — is the first in the hierarchy to yield a nonvanishing vertical component of $\mathbf{u}$. It was implicit in the pioneering work of Charney published in 1962, in a famous paper entitled "Integration of the primitive and balance equations". It starts with (2) and (4) but then adds to the resulting $\mathbf{u}$ field a horizontally irrotational correction field governed by another nonlinear elliptic boundary-value problem in the flow domain, a generalization of the "omega equation" previously developed by N. A. Phillips and others. The corrected $\mathbf{u}$ field is an asymptotically consistent improvement on (1), for small Rossby number, in the sense that it is one order more accurate in powers of the Rossby number. The boundary-value problem is derived by taking $\partial/\partial t$ of (4), then eliminating all the resulting time derivatives using the exact mass-conservation and vorticity equations and the inverse Laplacian of the vorticity equation. The vorticity equation expresses $\nabla^2(\partial\Psi/\partial t)$ in terms of diagnostically known, or knowable, quantities such as the corrected $\mathbf{u}$ field; so the inverse Laplacian is needed in order to eliminate $\partial\Psi/\partial t$ from $\partial/\partial t$ of (4).

This process of eliminating all the time derivatives has to be possible, in principle at least, if the end result is to be a balance relation, which by definition may not contain any time derivatives. When the elimination is carried out explicitly, a rather complicated set of integro-differential equations results, containing Green's function integrals whose details depend on the geometry of the flow domain. It may therefore be computationally more convenient to work with a set of equations from which $\partial\Psi/\partial t$ has not been eliminated, but has been allowed to remain as an unknown that can, in principle, be eliminated. Then "$\partial\Psi/\partial t$", in quotes, so to speak, must be regarded not as the actual rate of change of Ψ but, rather, as an auxiliary variable — better described as a *diagnostic estimate* of the rate of change, which must be expected to differ, in general, from the actual rate of change of Ψ. To avoid confusion over this point a special notation is sometimes used, such as Ψ_1 for a diagnostic estimate of $\partial\Psi/\partial t$, Ψ_2 for $\partial^2\Psi/\partial t^2$, and so on.

The general form of the functional dependence defining a balance relation, assuming a balanceable mass field $\Phi(\mathbf{x},t)$, is

$$\mathbf{u}(\mathbf{x},t) \;=\; \mathbf{u}^{\mathrm{B}}(\mathbf{x};\Phi(\cdot\,,t)) \tag{5}$$

where it is again emphasized that no derivatives or integrals with respect to t may appear: it must be possible, in principle at least, to eliminate them all. Time t enters solely via the second argument $\Phi(\cdot\,,t)$ of $\mathbf{u}^{\mathrm{B}}$. The notation $\Phi(\cdot\,,t)$ follows mathematical convention and signifies nonlocal spatial dependence. In other words, the second argument of $\mathbf{u}^{\mathrm{B}}$ is the whole *function*, Φ of $\mathbf{x}$, over the whole flow domain at the given instant t — not merely the *value* of Φ at the single value of $\mathbf{x}$ to which the left-hand side of (5) refers. Such nonlocal functions are sometimes called “functionals”. Even the geostrophic relation (1) is enough to illustrate the point, though it involves nothing more than the behavior of Φ in the immediate neighborhood of $\mathbf{x}$ — more precisely, it involves enough about that behavior to permit the evaluation of the two horizontal derivatives. The Bolin–Charney balance relations, generalized or not, are fully nonlocal, as is plain from the occurrence of elliptic partial differential operators like ∇^2 and, implicitly or explicitly, the associated Green’s function integrals. To find $\mathbf{u}$ from Φ or vice versa, one has to solve elliptic partial differential equations in the flow domain, as already emphasized, implying for instance that the value of $\mathbf{u}$ at some position $\mathbf{x}$ will depend on values $\Phi(\mathbf{x}', t)$ at other positions $\mathbf{x}'$ well outside the neighborhood of $\mathbf{x}$.

The generalized Bolin–Charney balance relation is often accurate enough for practical purposes, such as observational data analysis and assimilation, and the initialization of the full dynamics for numerical weather prediction. Of fundamental interest, however, from a theoretical viewpoint, is the fact that the pattern of elimination of time derivatives can be extended even further, resulting in balance relations that are more accurate still. The ideas involved seem to have been first explored by K. H. Hinkelmann in the 1960s, in connection with the initialization problem, and were later approached from another direction, based on normal-mode expansions, by B. Machenhauer, F. Baer, J. Tribbia and others.

The most accurate balance relations can, in some circumstances, be far more accurate than values of parameters like the Rossby number might ever suggest; and this accuracy extends over a far wider range of parameter values than could reasonably have been expected *a priori* — with values numerically of order unity, and even greater, in some cases. This astonishing fact — discovered by W. A. Norton in the late 1980s, through ingenious numerical experiments — cannot be deduced by inspection of the momentum equations or other forms of the equations of motion. It involves great mathematical subtlety, and full understanding has yet to be achieved. Some insight has come from studies of a related phenomenon in classical aerodynamics, the weakness of aerodynamic sound generation or “Lighthill radiation”. Recent work at the cutting edge of this problem can be found in papers by O. Bokhove, O. Bühler, D. G. Dritschel, R. Ford, J. C. McWilliams, A. R. Mohebalhojeh, S. Saujani, T. G. Shepherd, J. Vanneste, D. Wirosoetisno, I. Yavneh and others, appearing in the literature from about 2000 onwards.

Among other things this recent work has provided a clear answer, in the negative, to a question posed in 1980 by E. N. Lorenz: *Could there be an exact balance relation?* Could there be unsteady stratified, rotating flows that evolve in such a way that freely-propagating inertia–gravity waves are completely absent? More precisely, is there a slow manifold within the full phase space that is indeed an invariant manifold of the full dynamics?

The answer in the negative has sometimes been viewed with surprise, perhaps because KAM theory has shown that there are springy-pendulum examples, and similar examples from other low-order dynamical systems, in which the corresponding question has a positive answer as emphasized in work by O. Bokhove, T. G. Shepherd and others. In dynamical-systems language, there are swinging modes that confine themselves to invariant manifolds in the form of "intact KAM tori". In such cases, the swinging motion of the pendulum evolves as if the length of the spring and the elevation of the bob were *exactly* slaved to each other.

But the negative answer, for atmosphere–ocean dynamics, is now very clear from various lines of argument beginning with pioneering work of R. M. Errico and T. Warn, and strongly confirmed by the recent work mentioned above. It is also implicit in the nonlocalness, or action-at-a-distance, expressed by (5). Information (about chaotic vortex motion for instance) cannot in reality travel infinitely fast. Related to this is the fact that Lighthill radiation, though often exceedingly weak (accounting for the astonishing accuracy found by Norton) is almost always nonzero. In the atmosphere–ocean context, this says that unsteady vortical flow almost always radiates sound and inertia–gravity waves, though often very weakly. This in turn relates to dynamical-systems concepts such as Poincaré's "homoclinic tangle" and the breakup of KAM tori into thin "chaotic layers" or "stochastic layers". Lighthill's ideas make it overwhelmingly probable, even though not yet proven rigorously, that the so-called "slow manifold" is such a stochastic layer. Though astonishingly thin in places — over a far wider range of parameter values than could reasonably have been expected *a priori* — it is not a manifold, which by definition is infinitesimally thin. Though astonishingly accurate in some circumstances, the concept of balance is inherently and fundamentally approximate. The layer is sometimes referred to, therefore, as the "slow quasimanifold".

(Arguably, a self-contradictory term like "fuzzy manifold" is best avoided. By its mathematical definition a *manifold* is a perfectly sharp, smooth hypersurface and not at all fuzzy. Thus "fuzzy manifold" would add yet another item to the list of self-contradictory terms like "variable solar constant" and "asymmetric symmetric baroclinic instability" — which of course we inevitably have to live with but, perhaps, need not add to.)

One of the most peculiar manifestations of slow-quasimanifold fuzziness is the phenomenon sometimes called “schizophrenia” or “velocity splitting”. This is a generic property of the most accurate “balanced models”.

Just as the swinging modes of the springy pendulum can be described in a simplified yet remarkably accurate manner by imposing a functional relation between spring length and bob elevation, vortical flows can be described by simplified “balanced models”, constructed by imposing a balance relation from the start. This forces a true slow manifold into existence. The initialization of such a model requires only a single scalar field to be specified, such as the mass field, or the potential-vorticity field in the generalized sense. This scalar field is sometimes called the “master” field or “master” variable of the balanced model, to which all other dependent variables are slaved. The model has only one prognostic equation, involving only one true time derivative, the rate of change of the master field — as distinct from the diagnostic estimates of time derivatives that may be hidden inside the definition of the balance relation (5), such as the diagnostic estimates $\Psi_1, \Psi_2, \ldots$ already mentioned.

A famous example of such a model is the “Bolin–Charney balanced model” or “Bolin–Charney balance model”, or “isentropic-coordinate balance equations”, so-called, in which either mass or potential vorticity can be taken as the master field. Both are advected by the velocity field determined via the generalized Bolin–Charney balance relation. Here, as implicitly above, the term “potential vorticity” is to be understood in its exact (Rossby–Ertel) sense, and is to be evaluated with the same velocity field, namely that given by the generalized Bolin–Charney balance relation.

Now the term “velocity splitting” refers to the fact, only recently noticed, that no balanced model more accurate than the Bolin–Charney model can have a single velocity field that advects both mass and potential vorticity, and from which the exact potential vorticity is evaluated. Paradoxical though it may seem at first, all such highly accurate balanced models have one velocity field to advect the mass, and another to advect the potential vorticity. At the highest accuracies, the two fields differ by only a tiny amount, but differ they must. Related to this is the fact, already mentioned, that diagnostic estimates such as $\Psi_1, \Psi_2, \ldots$ differ from true time derivatives such as $\partial\Psi/\partial t,\ \partial^2\Psi/\partial t^2, \ldots$ In all these respects the Bolin–Charney balanced model has turned out to be wholly exceptional.

Velocity splitting was first noticed for Hamiltonian balanced models constructed from the full dynamics by the method of R. Salmon. All such models exhibit velocity splitting, at all levels of accuracy, though in a slightly different sense: one velocity field advects mass and potential vorticity but another evaluates potential vorticity. As Salmon first showed in the 1980s, the models can be constructed in a systematic way by imposing the balance relation (5) as a constraint on the full dynamics within the Hamiltonian framework. Technically speaking, the crucial step that produces a balanced model while preserving Hamiltonian structure is to restrict the "symplectic 2-form" of the full dynamics (a mathematical object that can contain both the Hamiltonian flow in phase space and variations about it) to the submanifold in phase space defined by (5).

As Salmon pointed out, one of the reasons for using the Hamiltonian framework is that it provides control over conservation principles. The framework, properly applied, guarantees that the balanced model will fully respect the standard conservation principles for mass, momentum, and energy, as well as the material conservation (material invariance) of potential vorticity. However, there is a fundamental tension between accuracy and conservation. The most accurate balanced models cannot be expected to respect conservation, beyond the material invariance of potential vorticity. That is because they are trying to mimic vortical flows that in reality produce Lighthill radiation, which involves wave-induced local mass rearrangement, and wave-induced fluxes of energy and momentum, none of which can be exactly described by the balanced model. It is therefore arguable that the most accurate balanced models will, by that very fact, not respect the standard conservation principles for mass, momentum, and energy. One cannot have both accuracy and conservation. Something has to give way.

Within the Hamiltonian framework, which automatically preserves the conservation principles, what gives way is the concept of a unique velocity field. Less obviously, the same thing happens with non-Hamiltonian balanced models of the highest possible accuracy — essentially because the neglect of Lighthill radiation still implies an imperfect representation of local mass rearrangement. This becomes noticeable, even with a non-Hamiltonian balanced model, as soon as one is computing with enough accuracy to see the fuzziness of the slow quasimanifold.

Note on terminology: The reader is warned that the term "geostrophic balance" and its shorthand form, "geostrophy", are sometimes used in the literature to mean balance more accurate than geostrophic, i.e. more accurate than (1). A common example is the phrase "geostrophic adjustment", which refers to the mutual adjustment of the mass and velocity fields to approach balance or to stay close to balance — and "balance" of course, in real fluid flow, means not (1) but the most accurate possible balance of the form (5). The example of a circular vortex adjusting toward ageostrophic, gradient-wind balance while radiating inertia–gravity waves is enough to illustrate the point. Gradient-wind balance is the particular case of Bolin–Charney balance that applies to a steady circular vortex. For the circular vortex it holds exactly when f is constant, and is equivalent to (1) plus a correction term representing relative centrifugal force. Thus by implication we have another piece of self-contradictory terminology, "ageostrophic geostrophic adjustment", unfortunately well established.

It may also be noted that the term "adjustment" is itself used in two distinct senses that are sometimes confused with each other. The first is "Rossby" or "initial-condition" adjustment, the mutual adjustment of the mass and velocity fields toward balance that occurs primarily because a system is started in an unbalanced state, an extreme example being the thought experiment described above. The second is "spontaneous" adjustment, the continual mutual adjustment of the mass and velocity fields to *stay close* to balance in unsteady vortical flow, even after initial conditions are forgotten. This second process, a far more subtle one, is the process that produces Lighthill radiation. It sets the ultimate limitations of the balance concept itself. For all the foregoing reasons, some authors are beginning to avoid the term "geostrophic adjustment", instead using the terms "Rossby adjustment" or "spontaneous adjustment" as appropriate. Lighthill radiation may also be referred to, therefore, as the "spontaneous-adjustment emission" of sound and inertia–gravity waves by unsteady vortical flows.

See also

Buoyancy and Buoyancy Waves: Theory. Coriolis Force. Dynamic Meteorology: Waves, Potential Vorticity. Hamiltonian Dynamics. Instability: Inertial Instability; Symmetric Stability. Kelvin–Helmholtz Instability. Quasi-geostrophic Theory. Teleconnections. Vorticity. Wave Mean-flow Interaction. Weather Prediction: Data Assimilation.

Further Reading

McIntyre, M. E. and Norton, W. A. (2000) Potential-vorticity inversion on a hemisphere. *J. Atmos. Sci.,* **57**, 1214–1235. *Corrigendum* **58**, 949. (Section 7 describes the only available investigation of a fundamental issue neglected above — how to make (5) Galilean invariant.)

Norbury, J. and Roulstone, I. (eds.) (2002) *Large-scale Atmosphere–Ocean Dynamics: Vol. II: Geometric Methods and Models*. Cambridge, University Press, 364 pp. (This book, just published, is an up-to-date reference on the mathematical aspects of balanced models, especially Hamiltonian balanced models, including a thorough discussion of the springy pendulum by P. Lynch.)

Saujani, S. and Shepherd, T. G. (2002) Comments on “Balance and the Slow Quasimanifold: Some Explicit Results”. *J. Atmos. Sci.*, **59**, 2874–2877. (This is a key to the recent literature on the accuracy of the balance concept.)

Warn, T., Bokhove, O., Shepherd, T. G. and Vallis, G. K. (1995) Rossby number expansions, slaving principles, and balance dynamics. *Q. J. Roy. Meteorol. Soc.*, **121**, 723–739. (This focuses on asymptotic expansions as one approach to finding high-order versions of (5).)