Turbulence without cascades – new insights from Jupiter's unearthly jets

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Abstract

A new idealized model of Jupiter's weather-layer dynamics gives fresh insight into the observed behaviour, and in particular into why Jupiter's jet system is so strikingly dissimilar to any terrestrial jet system. The model's behaviour is turbulent in the sense that the vortex interactions are nonlinear and manifestly chaotic, but not turbulent in the classic Kolmogorovian sense because, although energy is transferred upscale, from small-scale forcing to the jet-spacing scale, the transfer is not via a cascade but is direct. That is, the transfer is not via successive vortex-merging events and is not stepwise in wavenumber space. On the contrary, it goes upscale in a single step, via a completely different mechanism: the systematic migration of small vortices away from their generation sites. Vortex merging has no significant role. The model's vortex-generation sites correspond, approximately, to the cyclonically-sheared 'belts' on the real planet where moist convection is observed to generate small vortices and other vortical structures, sometimes accompanied by visible thunderheads and lightning. This is work with my former student Stephen I. Thomson, as reported in a recent publication (Thomson & McIntyre 2016) of which reprints are available at http://www.atm.damtp.cam.ac.uk/people/mem/nasa-cassini-index.html together with supplementary movies.

1. Motivation and model formulation

The traditional Kolmogorovian cascade models of two-dimensional turbulence, going back to the pioneering work of Batchelor, Kraichnan, and Rhines, have greatly influenced the community studying the great jetstreams observed in Jupiter's weather layer. For one thing, it is often taken for granted that, when energy is injected at small scales by moist convection, the large-scale energy of the jet system is supplied through the traditional upscale energy cascade, that is, stepwise in wavenumber space. However, the observed behaviour of the real planet has forced a reappraisal of these ideas.

Such a reappraisal is undertaken in Thomson & McIntyre (2016), hereafter TM16. It was motivated by three conflicts between the real behaviour and properties of the traditional 2D turbulent models, especially when those models are extended to allow finite values of the Rossby deformation length L_{D} ($= c/f$ where c is a long gravity-wave speed and f is the Coriolis parameter). Finite L_{D} is much more realistic than the infinite L_{D} that is often assumed following Batchelor, Kraichnan, Rhines, and others, implying strictly nondivergent 2D motion. The need for finite L_{D} and slightly divergent 2D motion is well recognized by the community working on Jupiter. It allows for the flexible interface between the weather layer and the much deeper dry-convective layer beneath (Figure 1).

One of the abovementioned conflicts arises from the straightness of the observed jets, which for the most part flow around latitude circles – literally straight in a Mercator

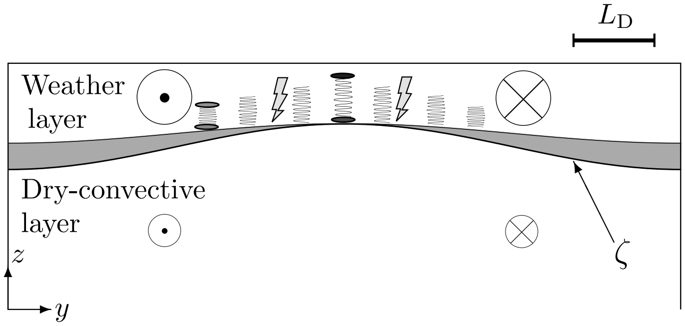

FIGURE 1. Schematic of the model setup and principal features, with y northward and z upward. The interface elevation ζ has an important role (see text). The model is quasigeostrophic and doubly periodic and is intended to idealize midlatitude conditions. The eastward and westward jets are strongest in the weather layer, consistent with the classic observational results of Dowling and Ingersoll and with the stability and straightness of the real jets; for further discussion see TM16. The bar at top right indicates an L_D value that allows stable jets and is in the range of realistic values. Cumulonimbus clouds are concentrated in the model belt (the central, cyclonically-sheared region), and are thought of as generating vortex pairs with cyclones below and anticyclones above. Such vortex pairs, called 'hetons' or 'heatons' in the oceanographic literature, can tilt and then propagate like ordinary two-dimensional vortex pairs. They are further simplified as ordinary vortex pairs in our idealized model. This is Figure 1 of TM16.

projection. There is no long-wavelength meandering. By contrast, traditional models with realistic L_D values produce jets that meander extensively, like the great jetstreams in the Earth's atmosphere and oceans. A second conflict is that forced-dissipative models within the traditional framework cannot attain statistical steadiness without artificial large-scale friction, usually taken as Rayleigh friction. Such friction is *highly* artificial because there is no solid surface anywhere near the bottom of the real planet's weather layer. A third conflict takes the form of an order-of-magnitude paradox regarding the energetics, explained in TM16 section 8.

An additional motivation for our work is a continuing need for idealized models. One might think that with today's computing power it would be better to focus on comprehensive 'general circulation models' that include all or part of the dry-convective layer. But for Jupiter there is an acute difficulty with such models, transcending the purely computational demands. At present we know far too little about where to place the lower boundary, and what lower boundary conditions to use. And unfortunately there is an extreme sensitivity of model behaviour to such boundary conditions. It arises from having a strong Taylor–Proudman constraint in the dry-convective layer. Wrong lower boundary conditions can give *qualitatively* wrong behaviour. This is further discussed in TM16. The boundary-condition uncertainties are due mainly to severe uncertainties in our knowledge of Jupiter's chemical composition and infrared radiative opacities.

Figure 1 and its caption summarize our model setup, which is doubly periodic and is intended to idealize midlatitude conditions while coming closer to the real planet, in significant respects, than traditional idealized models.

It is widely accepted in the Jupiter community that moist convection dominates the excitation of weather-layer turbulence. The real convection is small-scale and three-dimensionally turbulent and might well include supercell, tornado-alley-type phenomena, hidden from view below cloud-top. There are no observational or numerical studies that elucidate such details. *Faute de mieux*, therefore, we restrict ourselves to a highly idealized forcing of the 2D motion by injecting small vortex pairs at random times and locations. The vortex pairs are biased, in the sense that the cyclones are slightly weaker than the anticyclones. This is a radical departure from traditional forcings, which are always perfectly unbiased. And it makes a radical difference to model behaviour, in particular allowing us to dispense with artificial Rayleigh or other large-scale friction.

Other aspects of the model formulation are set out in TM16, along with the way in which this model resolves all three of the abovementioned conflicts. Crucial to such resolution, as it turns out, are two more aspects of the formulation. The first is to allow the forcing strength to increase with the interface elevation ζ. This is another radical departure from traditional forcings, which are always prescribed with spatially uniform statistics. However, such ζ dependence is a step toward realism because increasing ζ brings the interface closer to the lifting condensation level for water, in the real planet, favouring moist convection as suggested in Figure 1. There are several condensible chemical species, but the lifting condensation level for water is the lowest, and the most important in making the temperature profile subadiabatic or stably stratified near the bottom of the real weather layer.

The second aspect is to assume the presence of substantial zonal jets in the dry-convective layer, as indicated schematically in the lower part of Figure 1. Because the dry-convective layer is Taylor–Proudman constrained and enormously more massive than the weather layer, it is consistent to regard these deep zonal jets as prescribed steady flows. Their presence turns out to be crucial to stabilizing the weather-layer jets in the regimes considered, for realistic L_{D} values. Without them, shear instability would produce planetary-scale jet meanders, or disturbances even more violent, any of which would be conspicuous if they occurred on the real planet (TM16 sections 4b, 6f).

2. Energetics: the dominance of vortex migration

Figure 2 shows fields from a typical model run having the qualitative structure sketched in Figure 1. We count that structure as realistic because the central region, the model belt with its cyclonic shear, has a raised interface (positive ζ, solid contours in Figure 2a) and therefore stronger vortex-pair injections, the model's counterpart of stronger moist convection as observed in the belts of the real planet. Thanks to geostrophic and hydrostatic balance the interface is raised because, in this regime, the weather-layer jets are stronger than the deep jets. It is further raised, in a region of horizontal scale L_{D}, by the strong cyclone in mid-belt as shown by the closed contours in Figure 2a, centred on the biggest dark feature in Figure 2b. The grayscale in Figure 2b shows the model's quasi-geostrophic potential-vorticity field, q, darkest for cyclonic and lightest for anticyclonic values.

The small vortices move around chaotically, occasionally merging but more often eroding each other. This is a fully nonlinear vortex-interaction regime – hence 'turbulent' in the dynamical-systems sense – but there is no upscale energy cascade because the successive vortex-merging events that would mediate such a cascade are defeated, on average, by the attrition and eventual loss of small vortices by erosion. We went to some trouble to show that this was a genuine effect of the chaotic vortex interactions and that numerical dissipation had no significant role (Thomson 2015 and TM16 section 6a). A movie showing what happens is available from http://www.atm.damtp.cam.ac.uk/people/mem/nasa-cassini-index.html

The jets in the model's weather layer have been both sharpened and strengthened, relative to the deep jets, by the averaged eddy effects of the chaotic vortex dynamics. In this run the deep jets are prescribed with sinusoidal velocity profiles. The weather-layer jets are much sharper, with pointed velocity profiles, essentially because the model's background q field has become well mixed on either side of each jet, just as in the classical

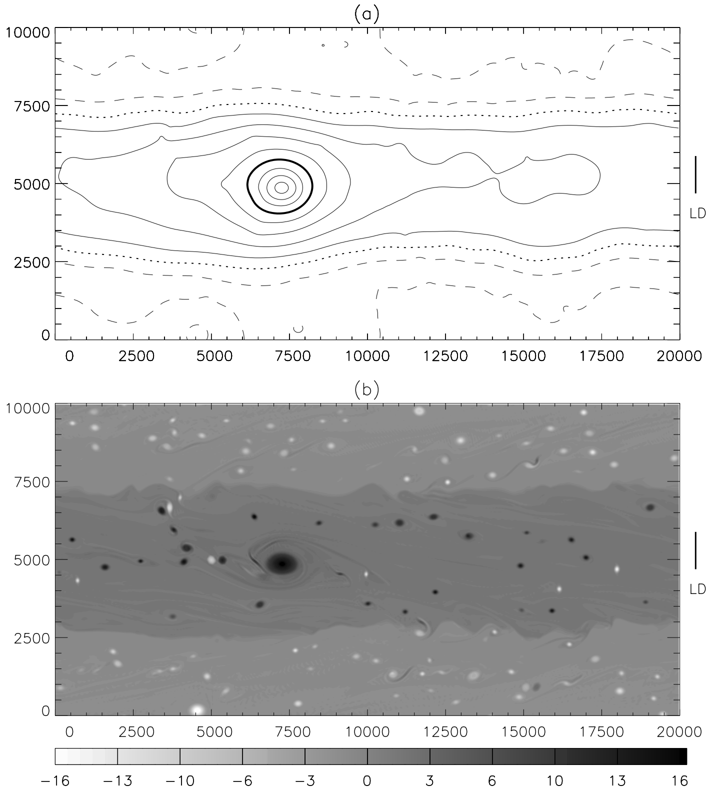

FIGURE 2. Snapshots of the flow fields in a typical run that has attained statistical steadiness and produced conditions qualitatively like those in Figure 1 (x and y axes in km). The value of L_{D}, 1200 km, is shown by the bars on the right. The top panel (a) shows the ζ field, with solid contours positive (interface raised, moist convection strengthened) and dashed contours negative (interface depressed, moist convection weakened). The bottom panel (b) shows the model's quasigeostrophic potential vorticity q (dark cyclonic, light anticyclonic). This is Figure 5 of TM16 (q.v. for further detail, and precise parameter values). Panel (b) corresponds to the first frame of the movie at http://www.atm.damtp.cam.ac.uk/people/mem/nasa-cassini-index.html

jet-sharpening process that forms terrestrial jetstreams. It shows up in Figure 2b as two sharp discontinuities between light and dark gray. However, the net strengthening of the weather-layer jets, essential to tilting the interface in the sense shown in Figure 1, depends on an entirely different process, namely the upgradient migration of small anticyclones out of the model belt into the anticyclonically-sheared regions on its periphery. Energetically, that is a direct transfer from small-vortex scales to jet-spacing scales.

Such upgradient migration, or *unmixing*, is helped by what is called the beta-drift or beta-gyre mechanism, as explained in TM16 section 6b.

REFERENCES

THOMSON, S. I. & MCINTYRE, M. E. 2016 Jupiter's unearthly jets: a new turbulent model exhibiting statistical steadiness without large-scale dissipation. *J. Atmos. Sci.* **73**, 1119–1141.

THOMSON, S. I. 2015 A new model of Jupiter's jet streams and the effects of moist convection. PhD thesis, University of Cambridge, 207 pp. The full text is available as a .pdf file from http://emps.exeter.ac.uk/mathematics/staff/sit204/