

**How much better** do we understand the dynamics of stratospheric warmings  
– **and what has it taught us about fundamental issues in geophysical and planetary fluid dynamics?**

Michael E. McIntyre,  
Dept of Applied Mathematics & Theoretical Physics,  
University of Cambridge, UK

Each question has the same quick answer, “**lots**”. Talk will try to make both more specific:

1. Re warmings as such: recent advances & challenges. **Whence linear theory?**
- 2a. Fundamental points with wider implications, e.g. countering some **myths about jets** on Earth, on Jupiter, and in tokamaks. The overarching **general-circulation paradigm change** (history in Section 1 of Dritschel & Mcl (2008, *J. Atmos. Sci.*). Role of **unconscious assumptions**.
- 2b. Fundamental points that **should be in the textbooks**, e.g. Kelvin’s circulation theorem  $\leftrightarrow$  “nonacceleration theorems” (shining exception: Bühler 2009)

Reprints, preprints & corrigenda: websearch **”lucidity principles”**  
then back to my home page at the strings “jets”, “DIMBO”, and “Rosenbluth”

convection  
zone

$2\pi\Omega/n\text{Hz}$

450

Schou et al '98

400

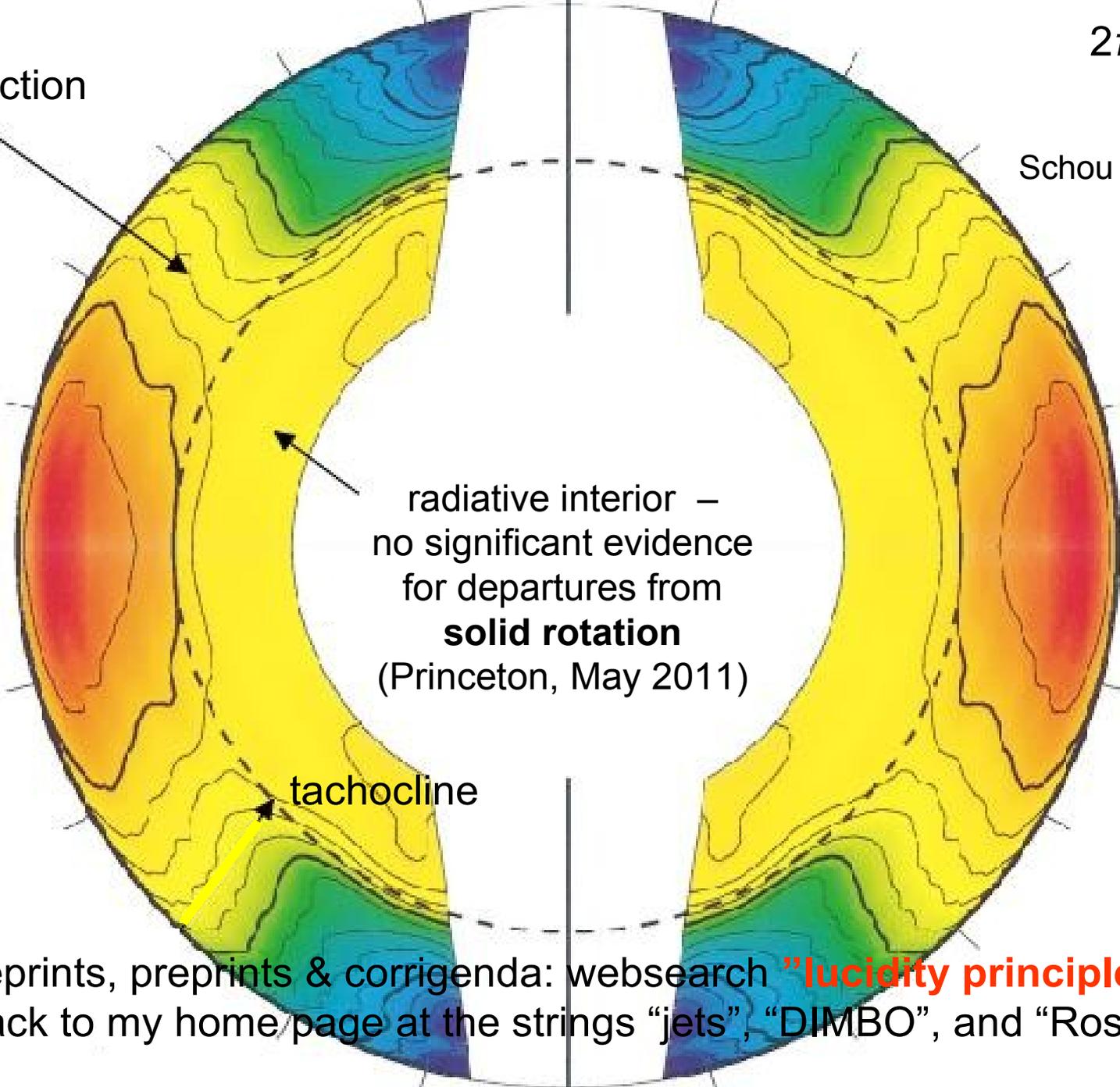
radiative interior –  
no significant evidence  
for departures from  
**solid rotation**  
(Princeton, May 2011)

350

tachocline

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300

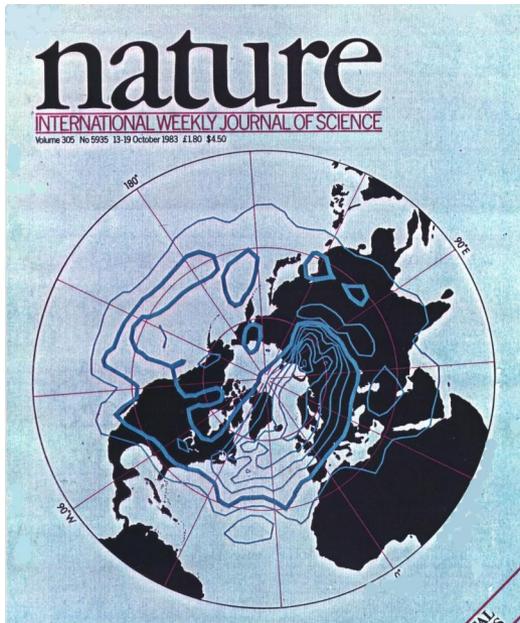


Historical snippet: **Here I quote myself *against* myself.** In the 1982 review I wrote that Rossby wave breaking and the consequent PV mixing had perhaps escaped notice because of

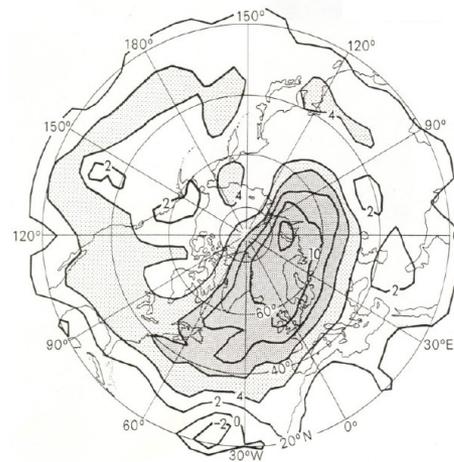
**“the near-impossibility of drawing isentropic maps of potential vorticity”** from data **“and thus seeing directly what is going on”**.

I think I'd taken too much notice of the words of an eminent and intimidating colleague – contrary to our Royal Society's motto *Nullius in verba*.

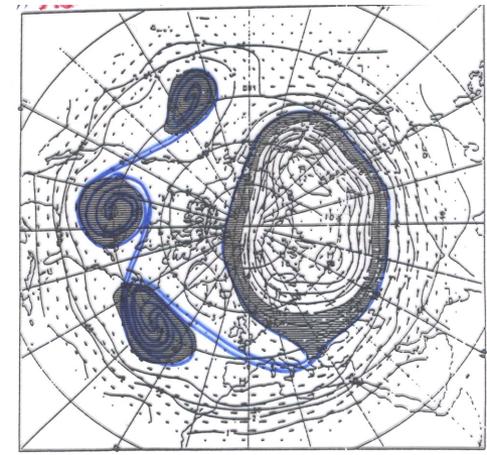
Tim Palmer, Alan O'Neill and co-workers at the UK Met Office quickly proved me, and the eminent colleague, wrong with a **“damn fool experiment”**, computing mid-stratospheric isentropic maps of PV from satellite data giving us **“a blurred view of reality seen through... knobbly glass”** of the **“world's largest breaking waves”**:



Corrected (1984, *J. Atm. Terr. Phys.*)



Clough et al 1985 (*Q. J.. Roy. Met. Soc.*)



The “damn fool experiments” created wider ripples, including new and deep insights into tropospheric cyclogenesis. **Here’s an insider’s view of the history:**

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– SSW better  
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The Reading **PV Song** 🎵

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( 😊 Cambridge GEFD  
Summer School is  
being resurrected!)

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Project Springfield:

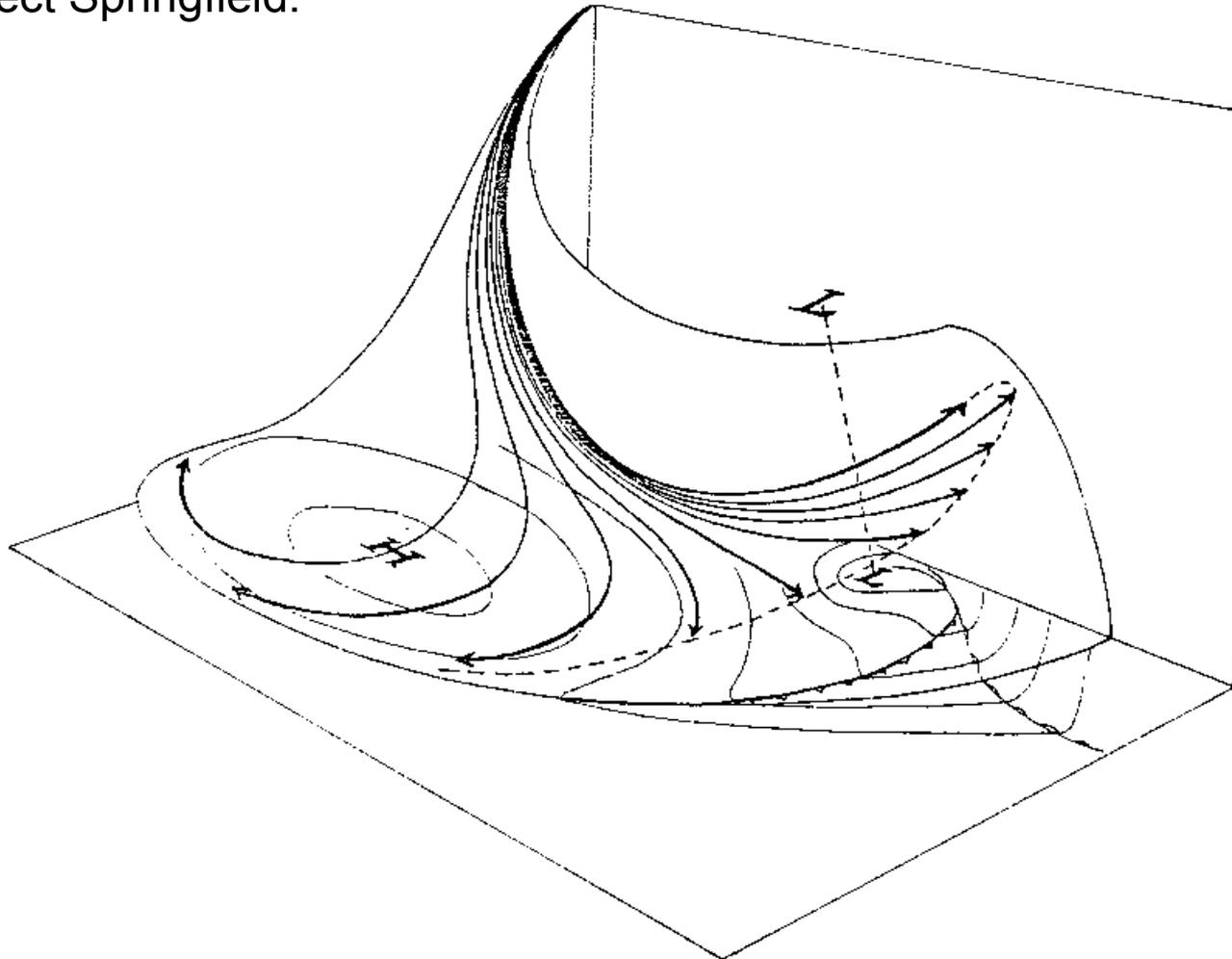


Figure 6 Trajectories of Extruded Stratospheric Air

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Not the world's largest  
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fundamentally similar.

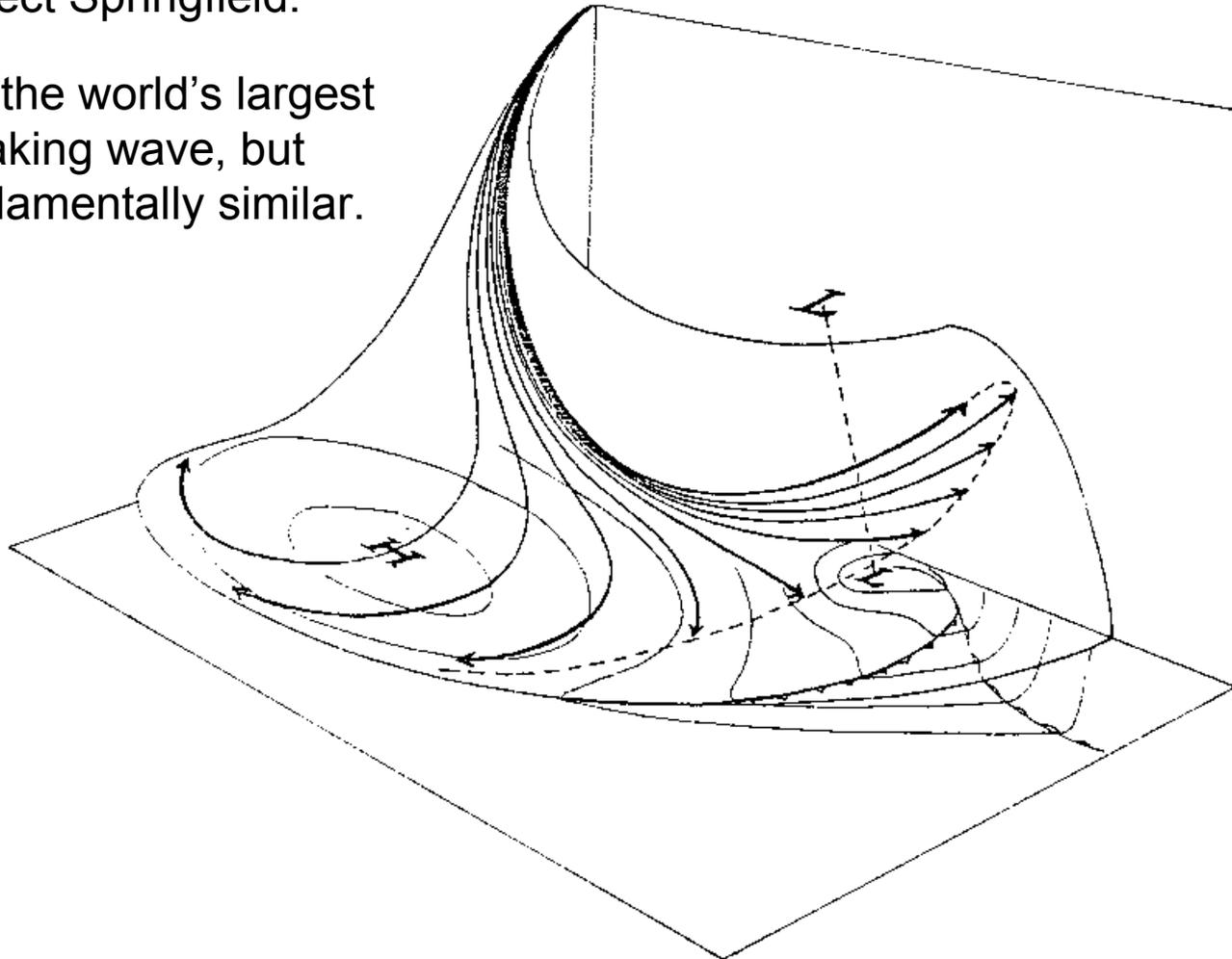


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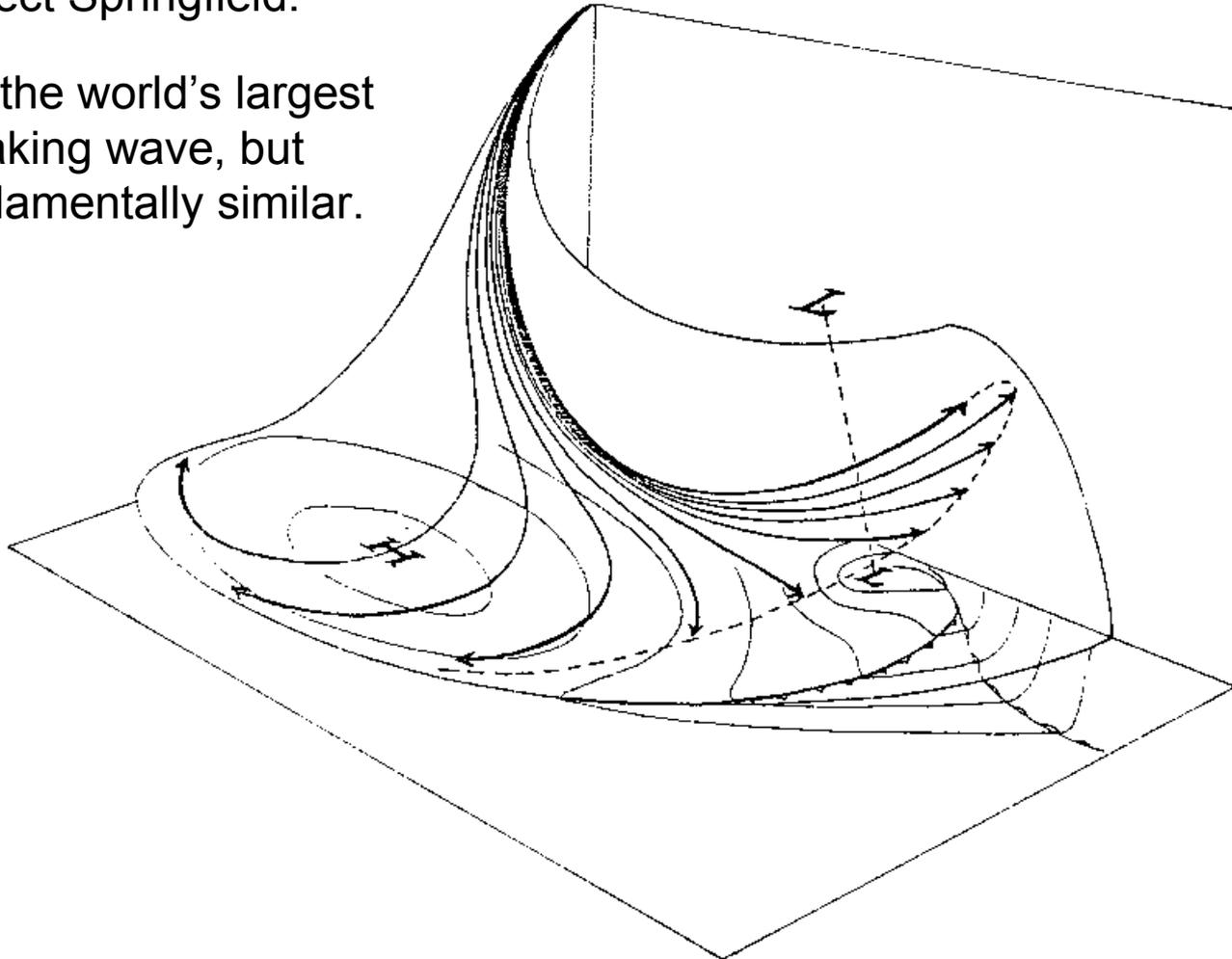


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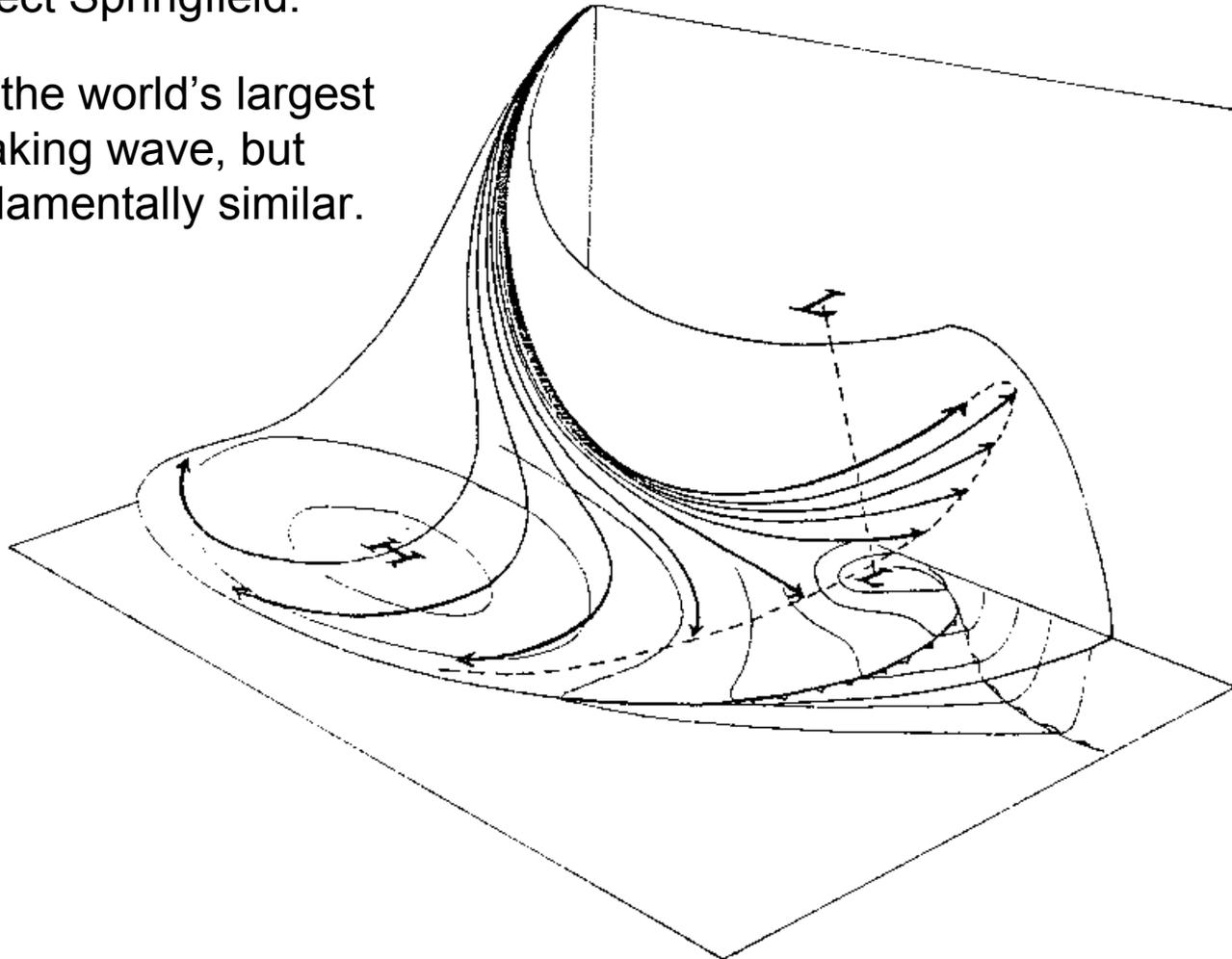
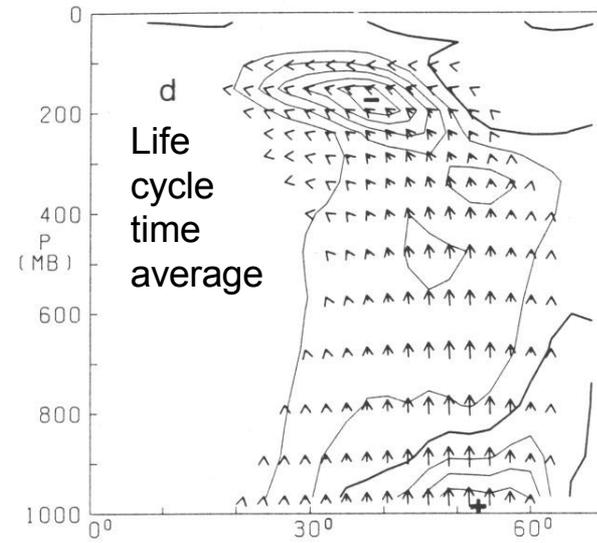
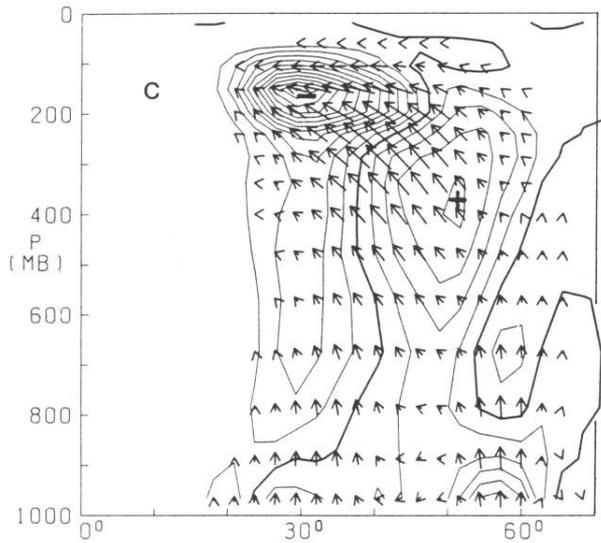
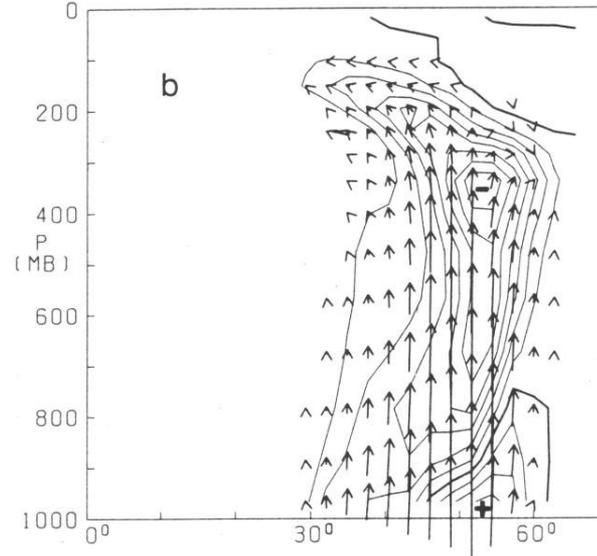
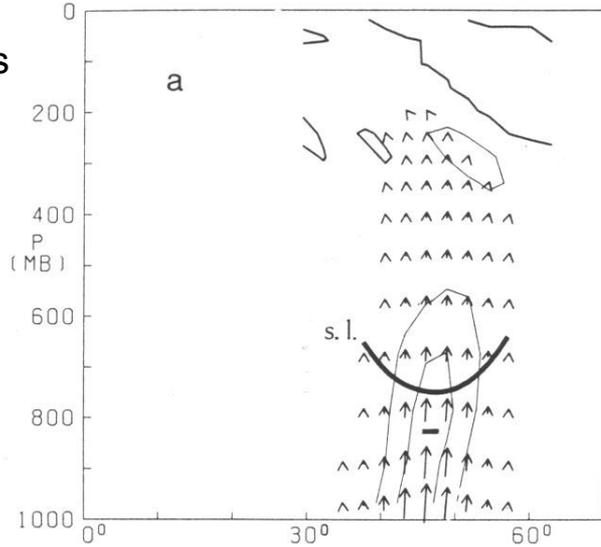


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Not only is wave breaking involved, but also, surprisingly, wave **propagation!**  
This was discovered accidentally in another “**damn fool experiment**”...

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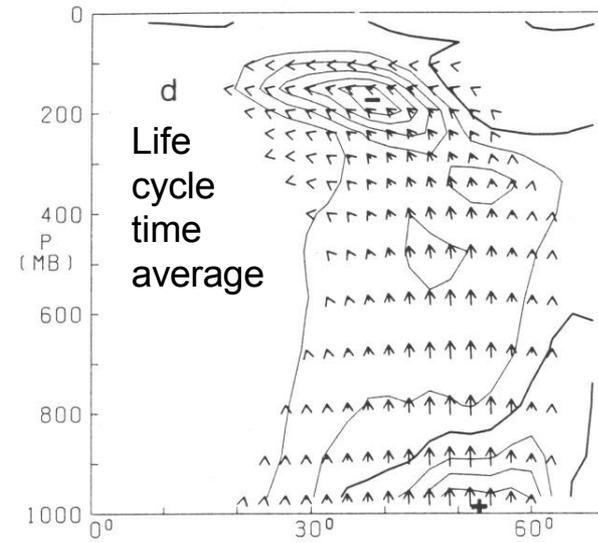
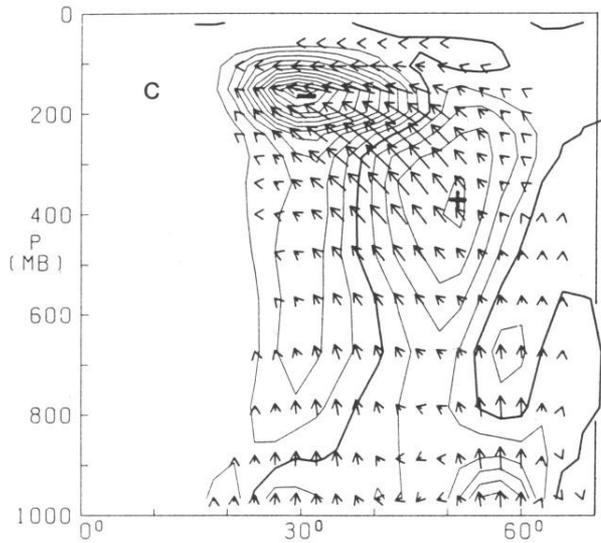
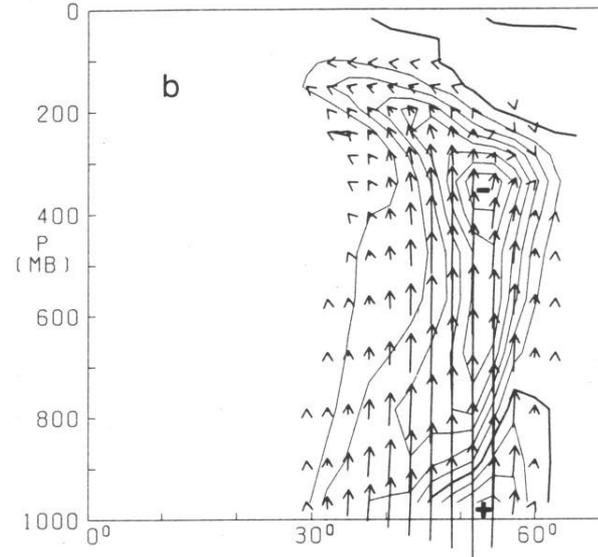
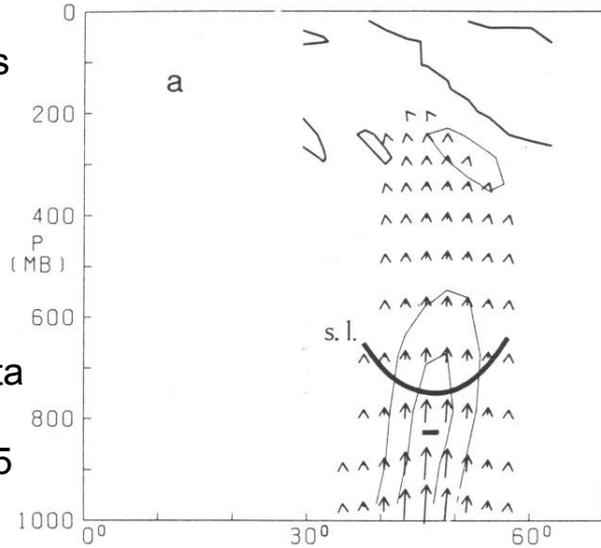
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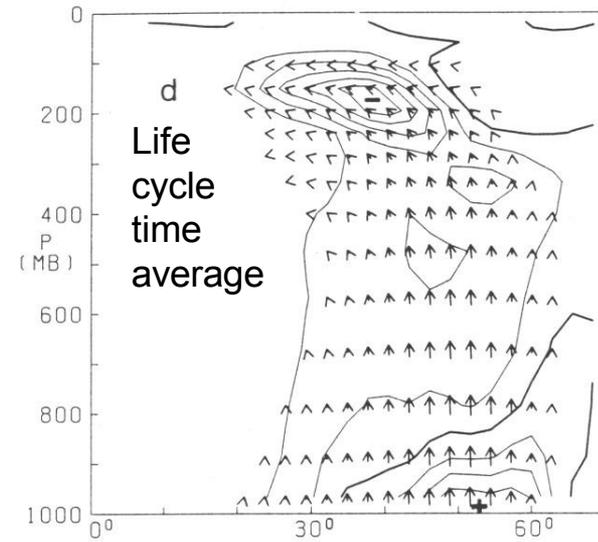
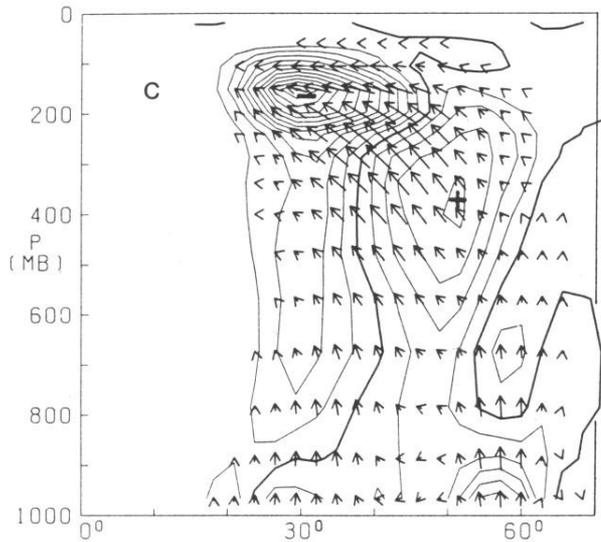
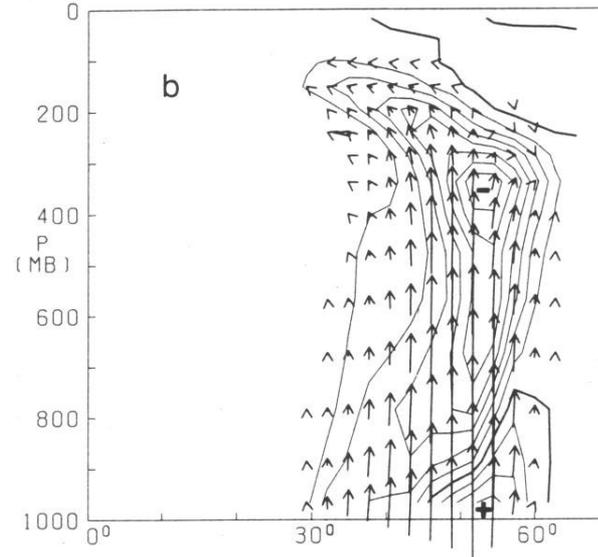
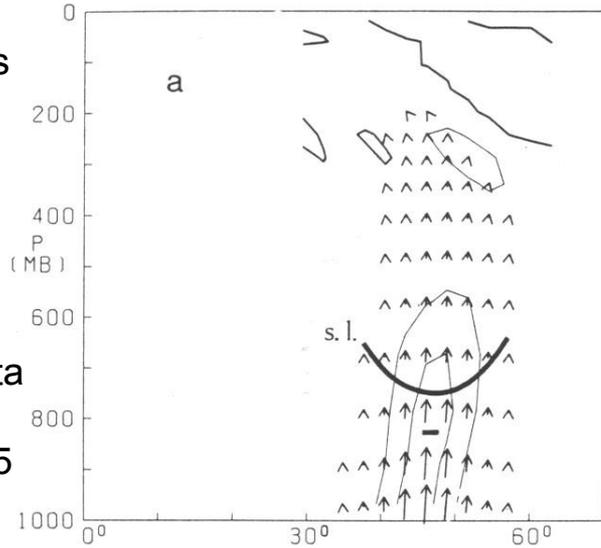
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maps see  
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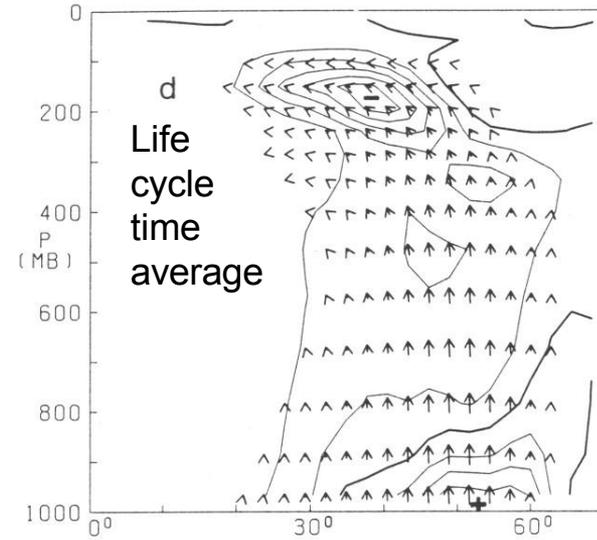
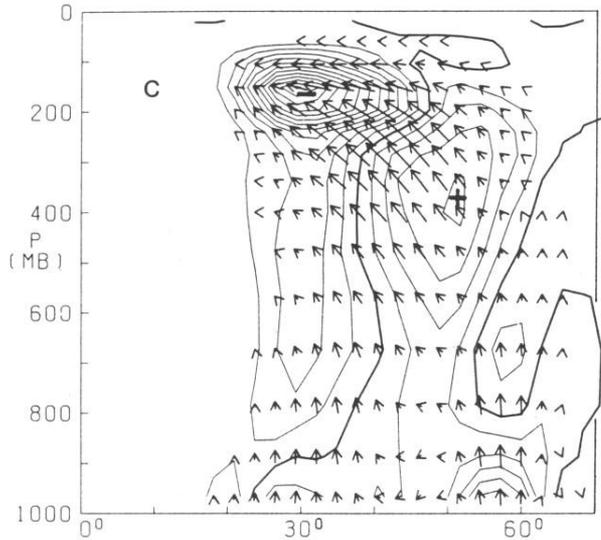
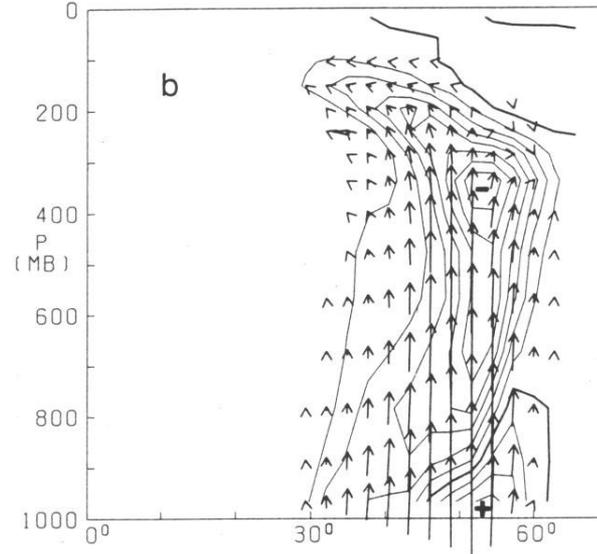
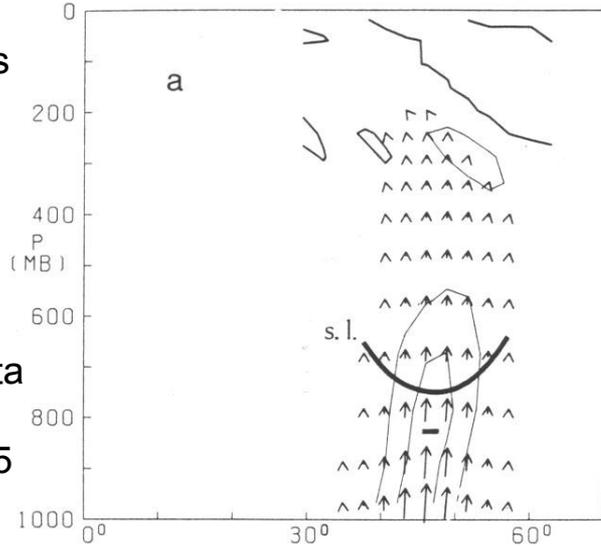


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The “saturation, propagation, saturation” pattern was a big surprise

– and it vindicated Dickinson’s seminal 1969 work (*JAS*), the first dawning of a complete, robust understanding of Starr’s “negative viscosity”. (History in Dritschel & Mcl. 2008).

Hindsight shows the foregoing as episodes within a **major paradigm change**:



Understanding began to emerge **after** V.P. Starr's 1968 "negative viscosity" book, beginning with ←Jule Charney, Ernst Kleinschmidt, Bob Dickinson.→ History long and tortuous – no "Einstein moment". But **today's understanding is crystal-clear.**



Historical review in Dritschel & Mcl (2008, *J. Atmos. Sci.*) on my home page.

**The paradigm change** (in our thinking about large-scale atmospheric dynamics, over the past century) can be summarized thus:

"turbulent atmosphere" (**frictional**)

→

"**radiation-stress**-dominated atmosphere" (**often anti-frictional**)

Accompanying insight: "there is no such thing as turbulence **without waves.**"

Indeed, it's now clear that the generic role of wave propagation mechanisms illustrates one of the grand themes of physics, the **dynamical organization of fluctuations** with systematic mean effects.

But whence this phrase "**damn-fool experiment**"?

“Erasmus Darwin held that every so often you should try a **damn-fool experiment**. He played the trombone to his tulips. This... result... was negative. But other... **impudent ideas** have succeeded...”

– *Littlewood’s Miscellany*, ed. Béla Bollobás



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(*E.g.*, via the relation between “nonacceleration theorems” and Kelvin’s circulation theorem.)

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... today the stratospheric “surf zone” is a hard-edged reality, familiar from **advanced remote sensing and high-tech weather forecasting**:

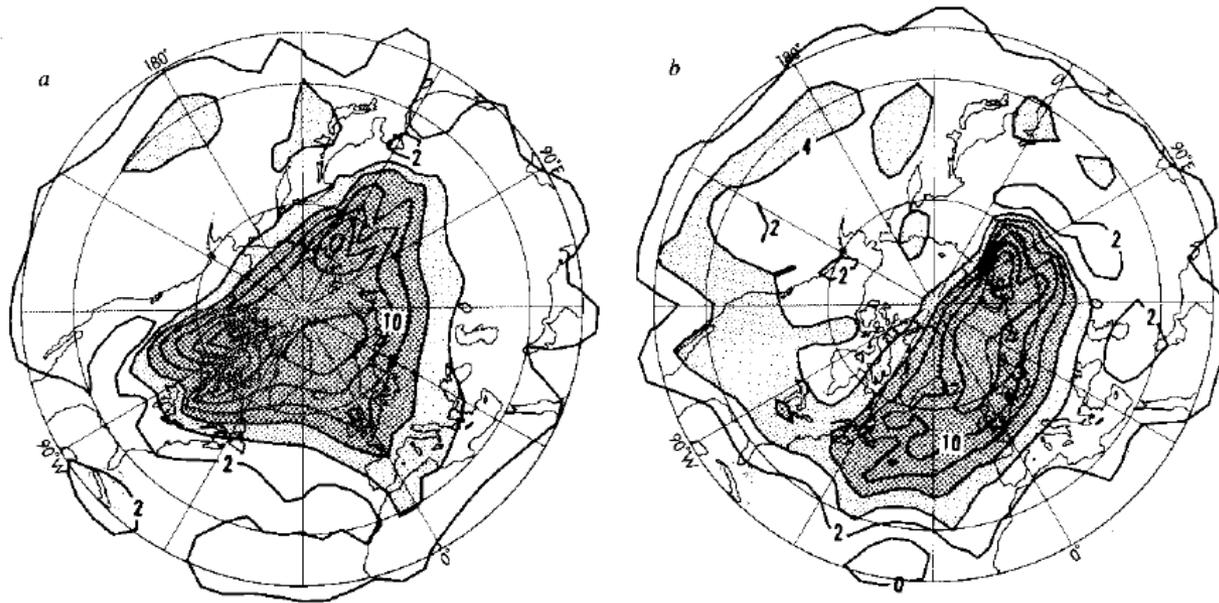
## Breaking planetary waves in the stratosphere

M. E. McIntyre\* & T. N. Palmer†

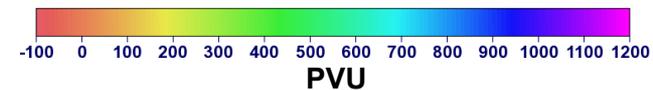
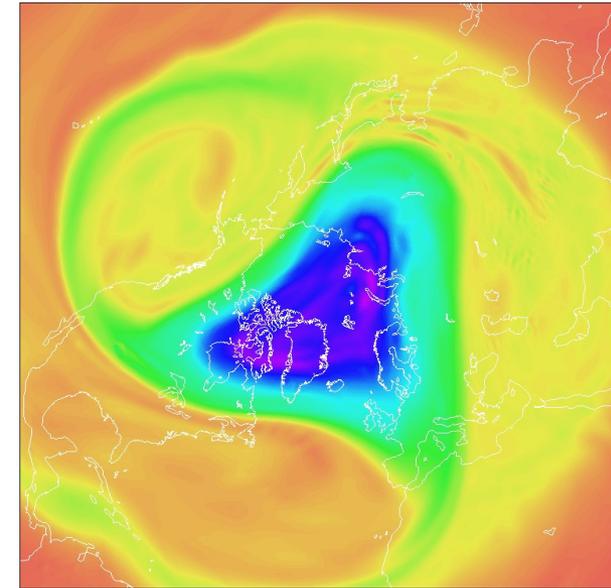
\* Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, UK

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Movie



Potential vorticity at 850K 00UTC 1979/01/17



**Fig. 2** Coarse-grain estimates of Ertel's potential vorticity  $Q$  on the 850 K isentropic surface (near the 10-mbar isobaric surface) on 17 (a) and 27 (b) January 1979, at 00 h GMT. The southernmost latitude circle shown is 20° N; the others are 30° N and 60° N. Map projection is polar stereographic. For units see equation (5) onwards. Contour interval is 2 units. Values greater than 4 units are lightly shaded, and greater than 6 units heavily shaded.

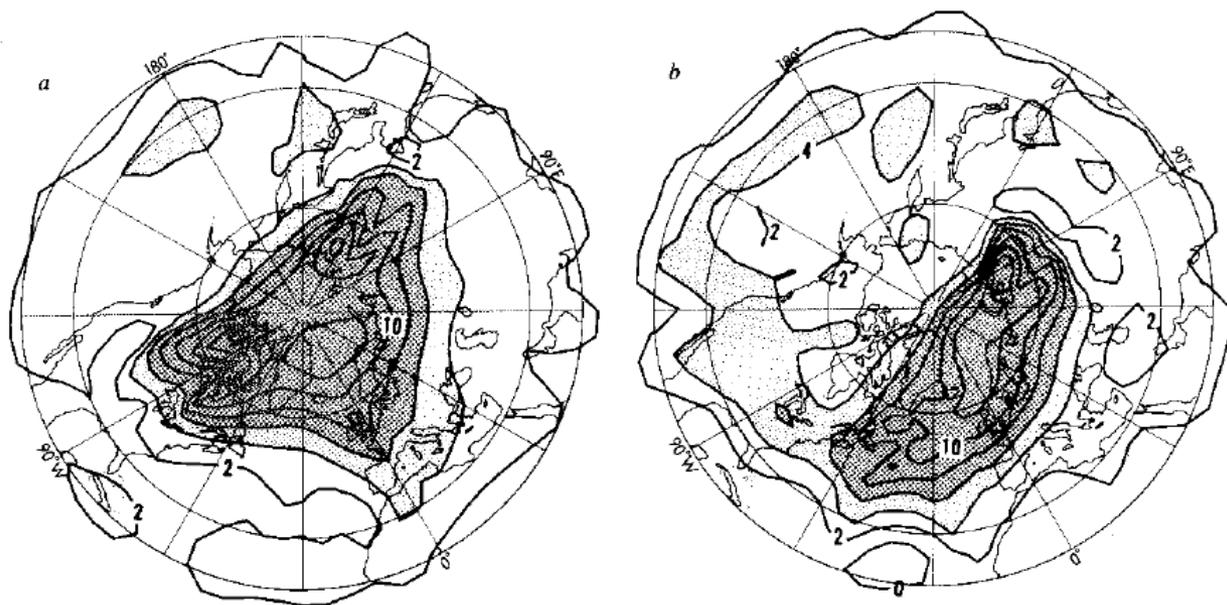
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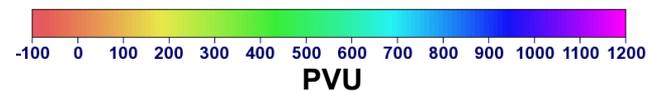
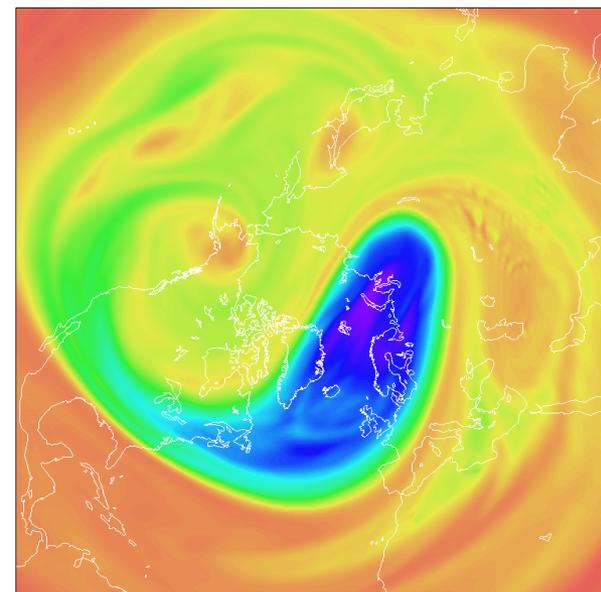
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Final state



Potential vorticity at 850K 00UTC 1979/01/27



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Courtesy Dr A J Simmons,  
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Range Weather Forecasts:

# McIntyre and Palmer (1983), revisited

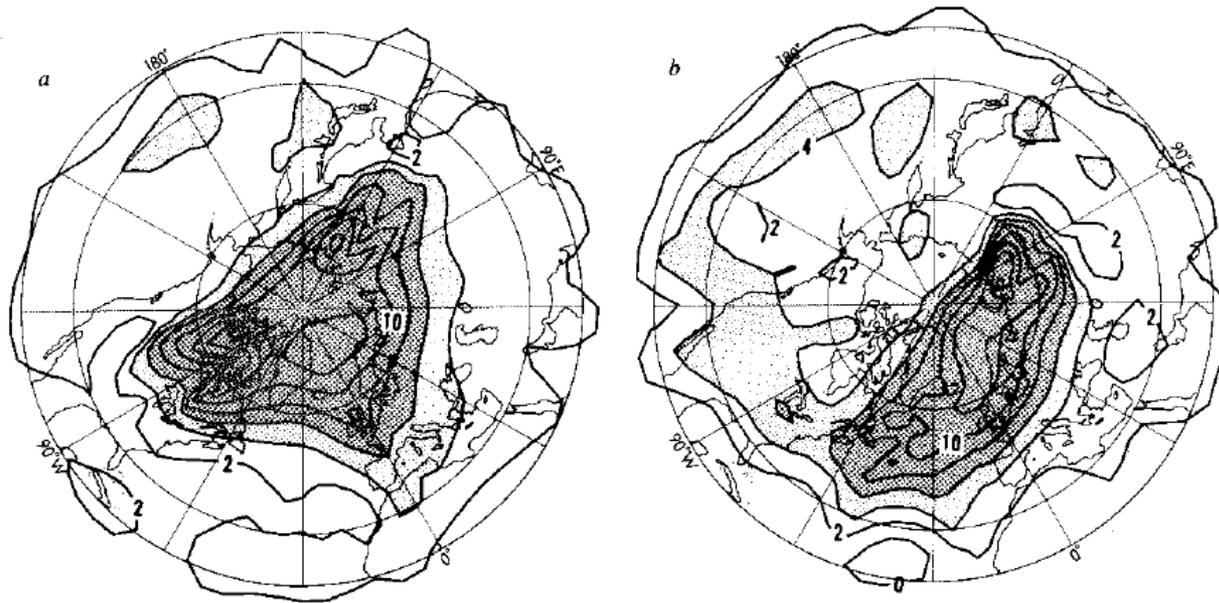
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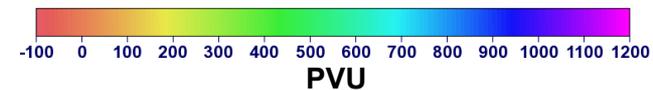
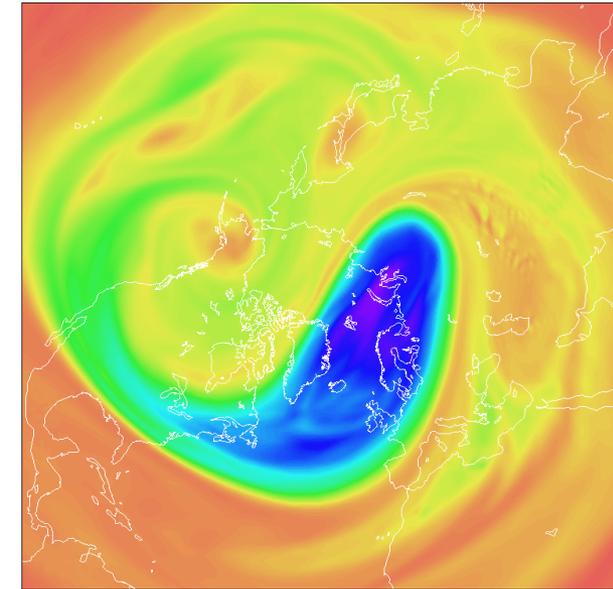
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Potential vorticity at 850K 00UTC 1979/01/27



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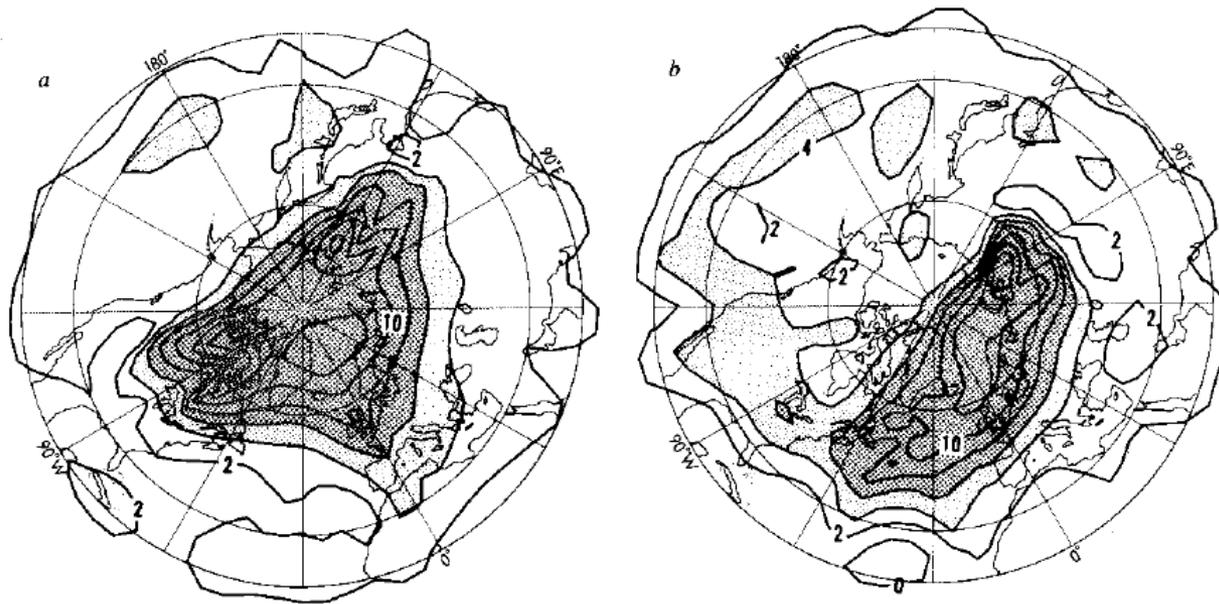
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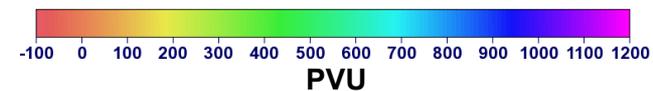
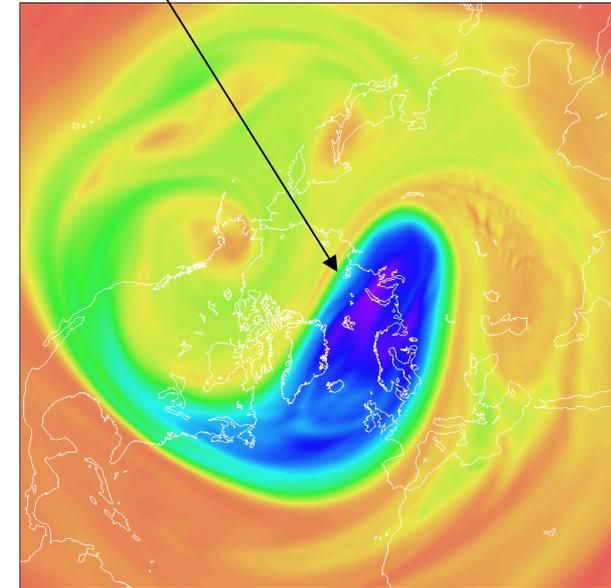
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Eddy-transport  
barrier

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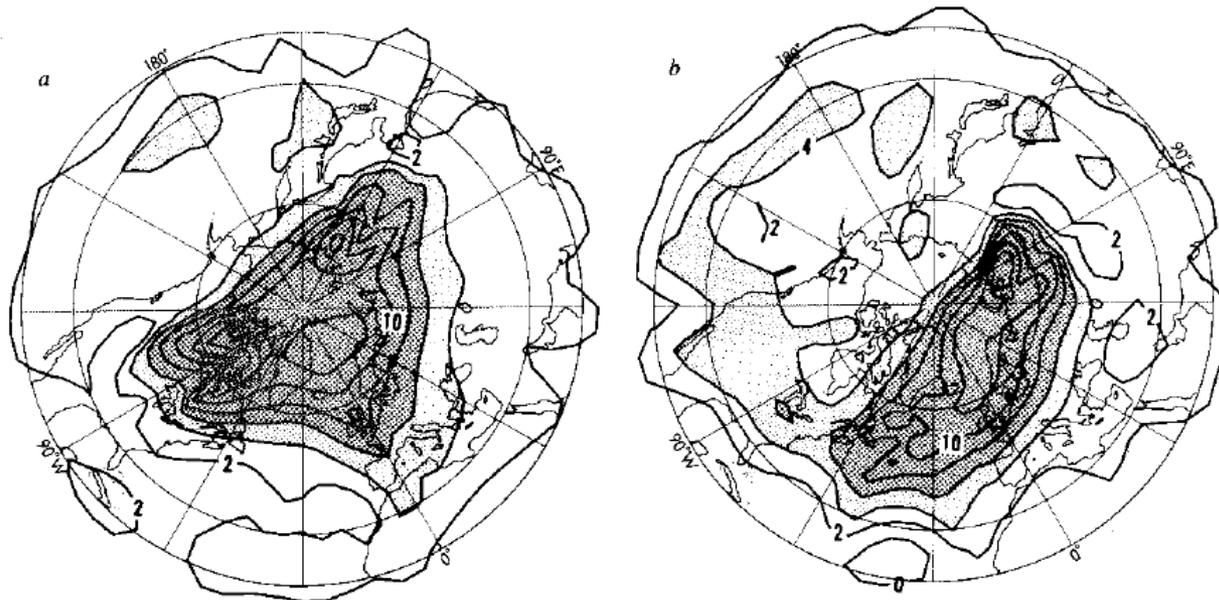
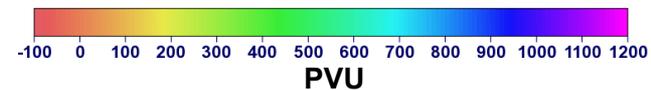
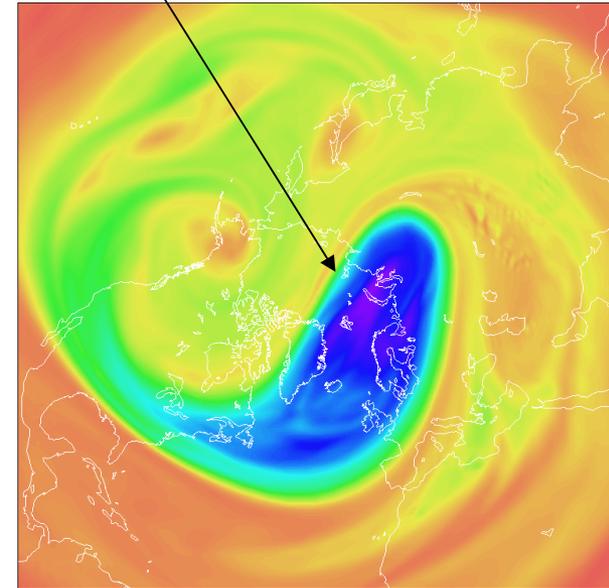
† Meteorological Office, Bracknell, Berks RG12 2SZ, UK

Our first view was indeed  
**blurred and knobby.**

Eddy-transport  
barrier

**This is nonlinear  
fluid dynamics!**

Potential vorticity at 850K 00UTC 1979/01/27



**Fig. 2** Coarse-grain estimates of Ertel's potential vorticity  $Q$  on the 850 K isentropic surface (near the 10-mbar isobaric surface) on 17 (a) and 27 (b) January 1979, at 00 h GMT. The southernmost latitude circle shown is 20° N; the others are 30° N and 60° N. Map projection is polar stereographic. For units see equation (5) onwards. Contour interval is 2 units. Values greater than 4 units are lightly shaded, and greater than 6 units heavily shaded.

Courtesy Dr A J Simmons,  
European Centre for Medium  
Range Weather Forecasts:

# McIntyre and Palmer (1983), revisited

## Breaking planetary waves in the stratosphere

M. E. McIntyre\* & T. N. Palmer†

\* Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, UK

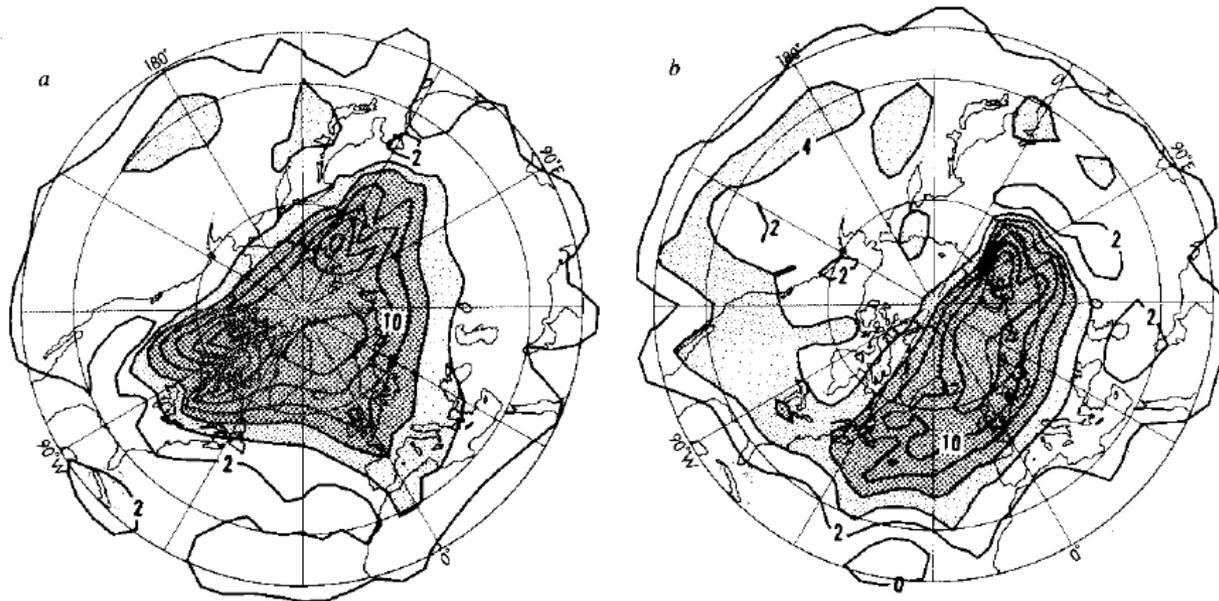
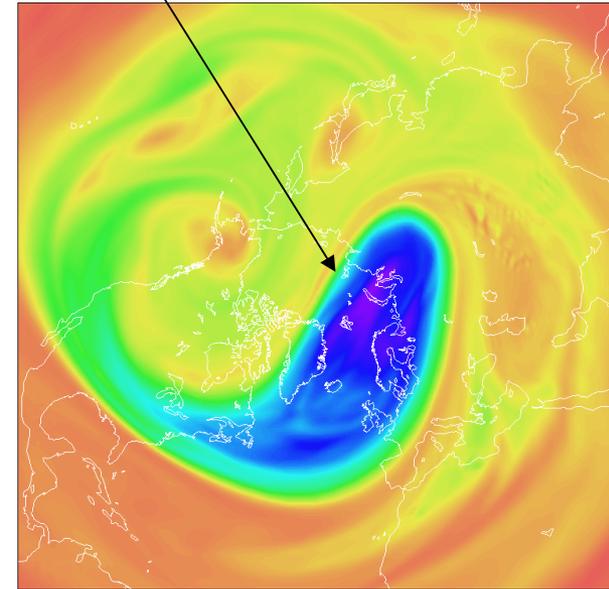
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So the apparent success of linear theory is a **profound conundrum!!**

**Why / when does the linear theory of planetary-scale Rossby waves do so much better than it ought?** (*Many papers following Matsuno 1970, J. Atmos. Sci.*)

This linear theory is **heavily relied on** in current thinking about the **annular modes** of variability – of **co-variability** of the stratosphere and troposphere – NAM and SAM –

– in particular, current thinking relies on the kind of linear behaviour studied in Chen and Robinson (1992, *J. Atmos. Sci.*) focusing on variability of the **Matsuno refractive index** near the subpolar tropopause.

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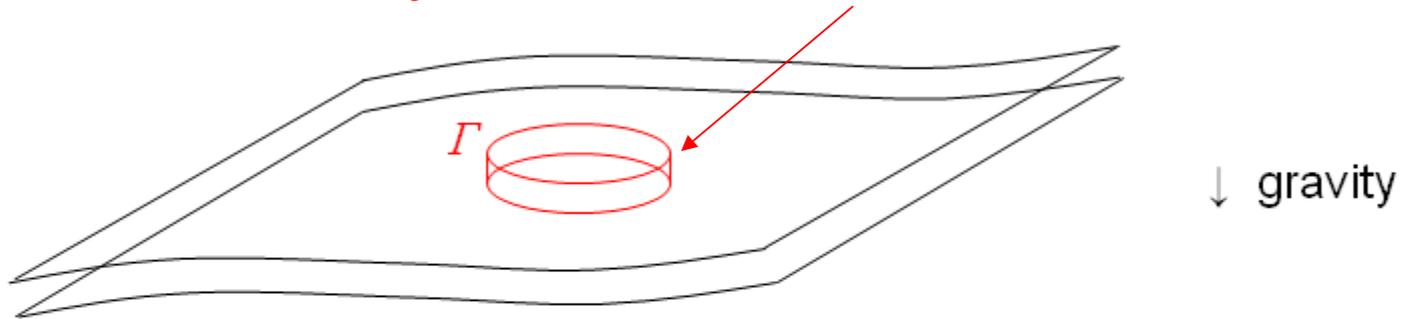
**I think the answer lies in PV fundamentals.**

A quick reminder of those fundamentals:

In both single-layer and multi-layer systems, we can define the PV as the suitably normalized Kelvin circulation of an infinitesimal material circuit  $\Gamma$  lying in a stratification surface.

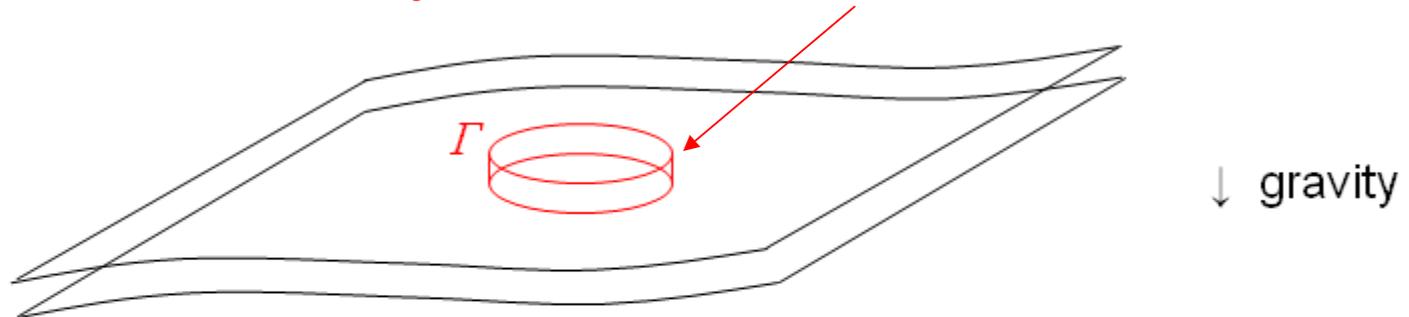
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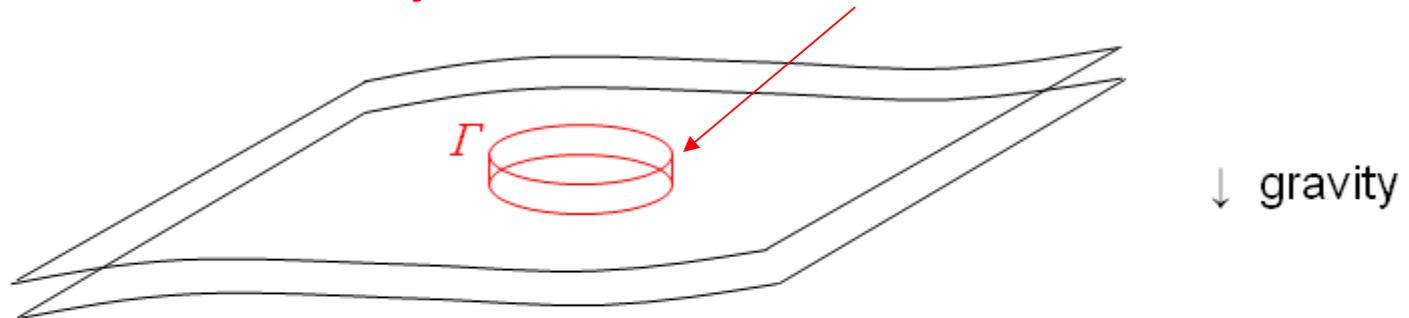
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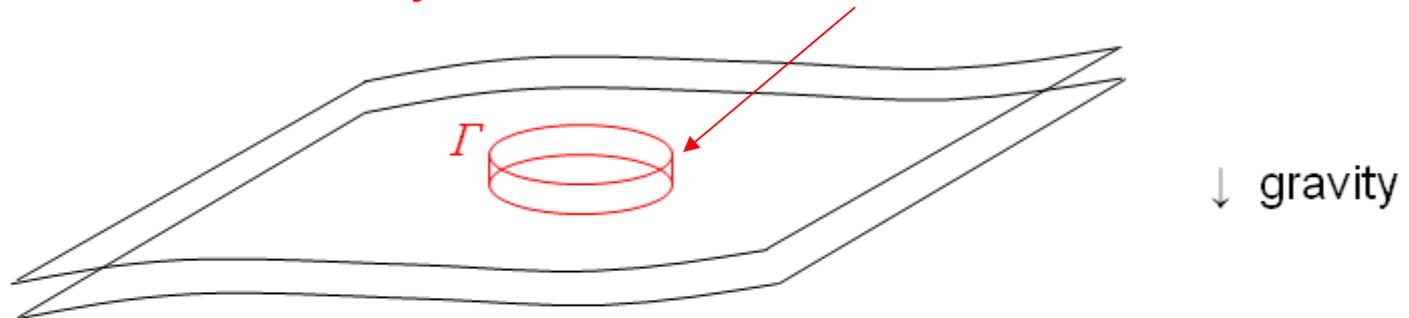


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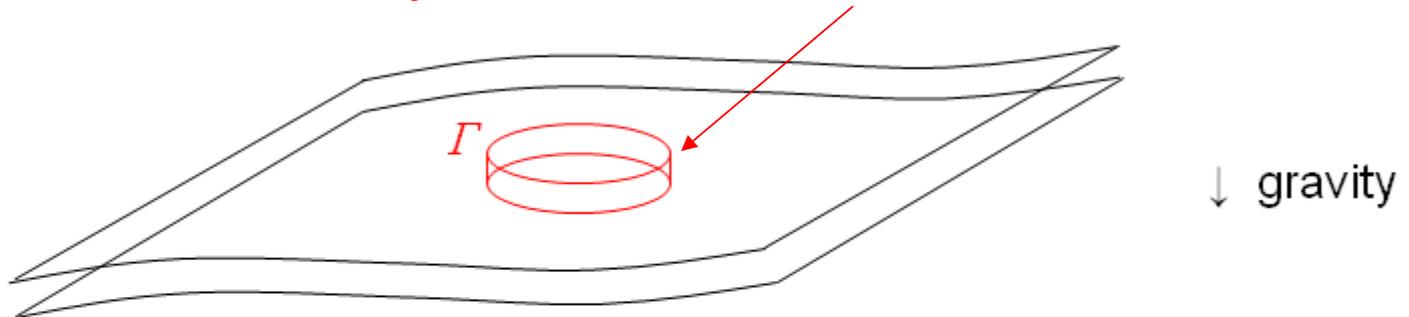


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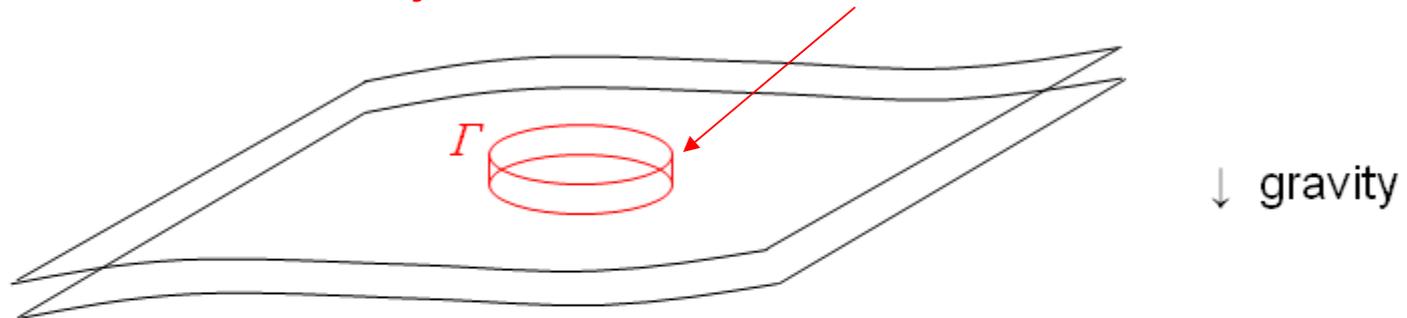


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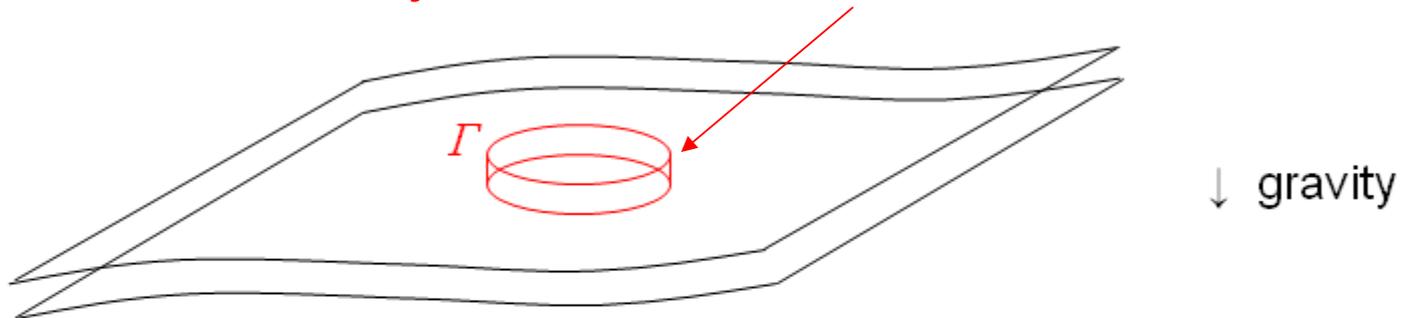
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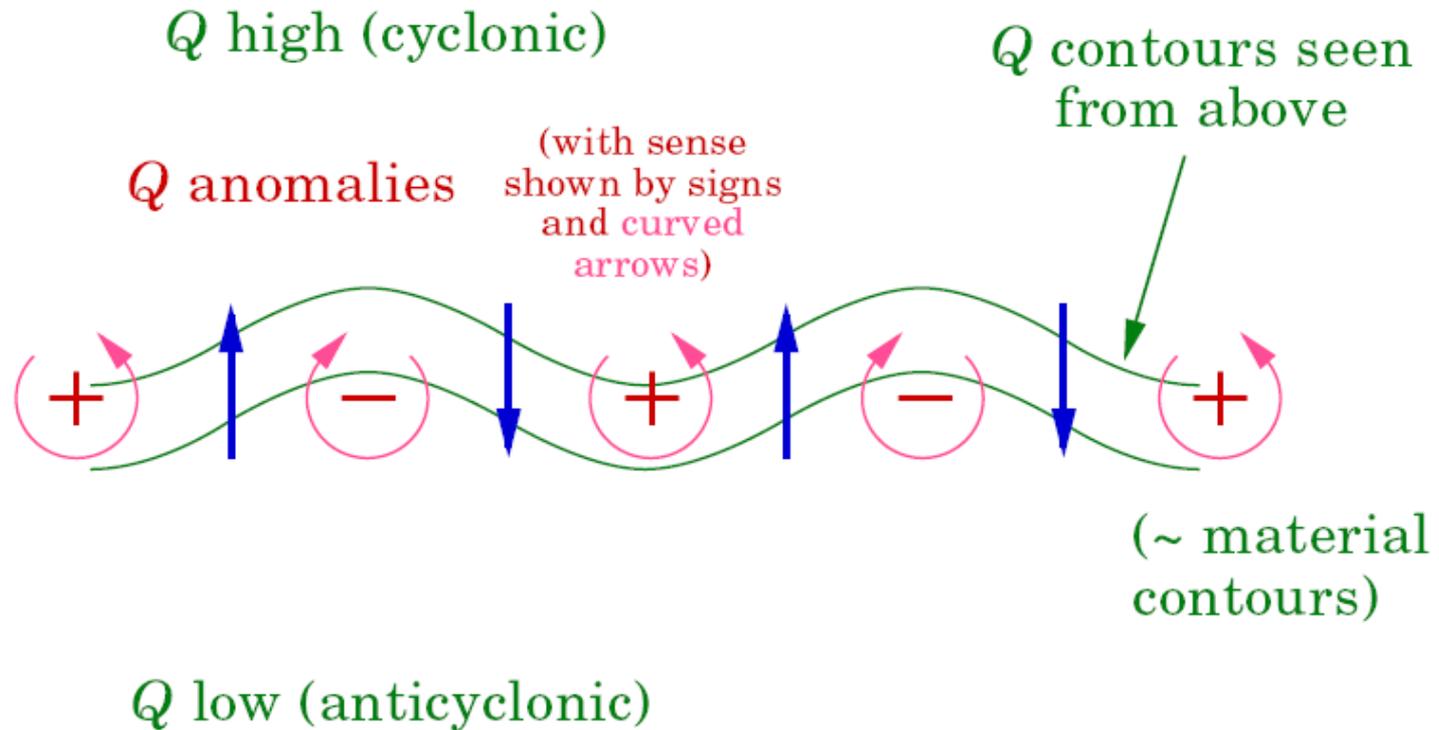
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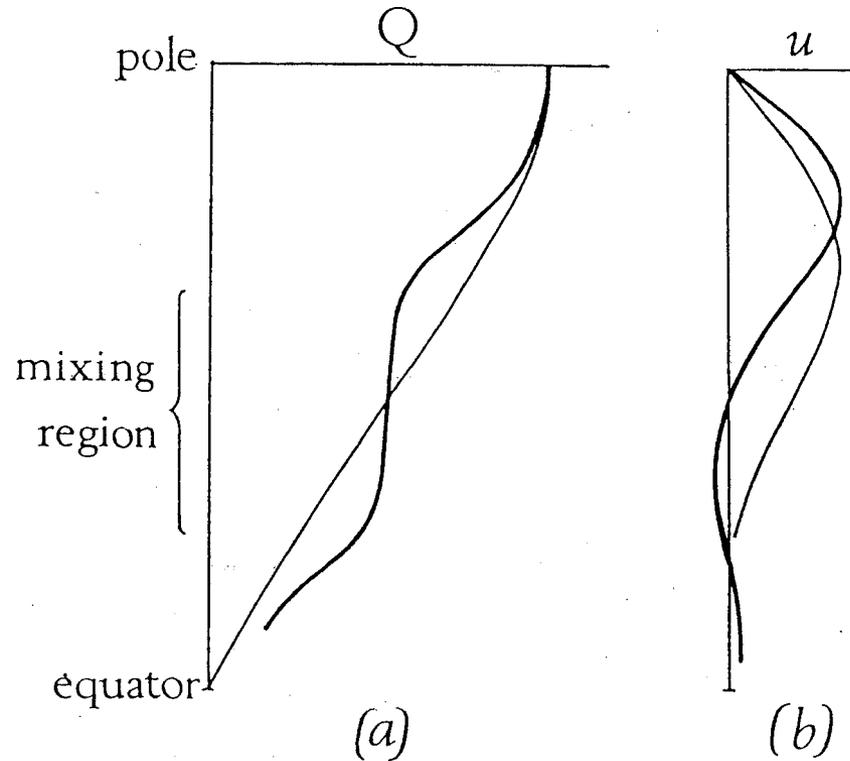
PV invertibility is seldom flagged up clearly in textbooks – even when explaining **Rossby waves!!**

# Rossby wave mechanism

( $Q$  is the PV)



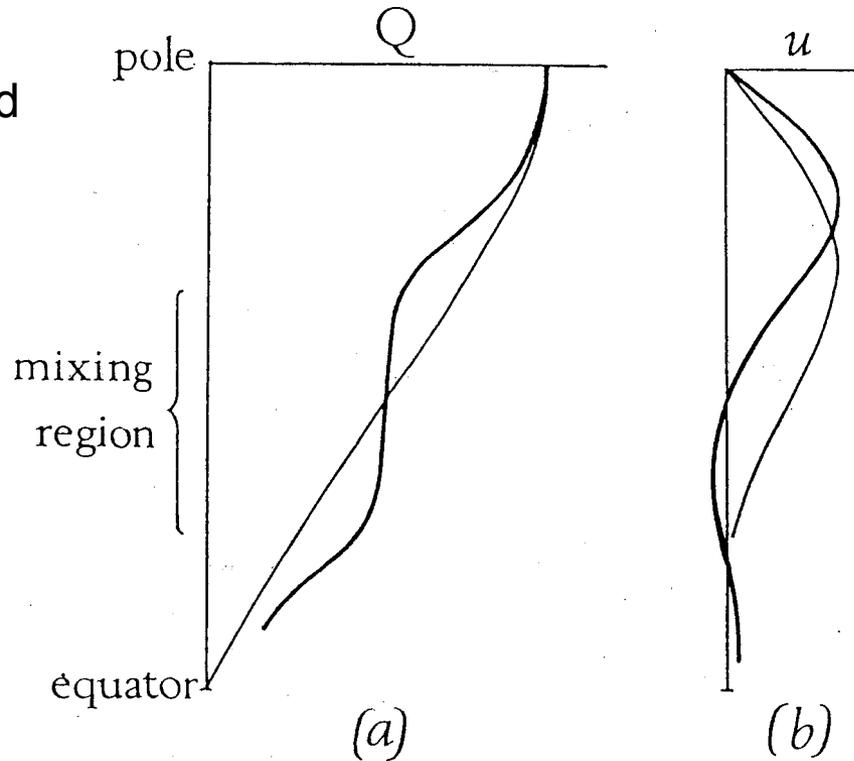
Notice how easily PV fundamentals solve what was mysterious to Starr and Lorenz, the great **negative-viscosity** or **jet-self-sharpening** conundrum:



(Fig 5 from  
my 1982  
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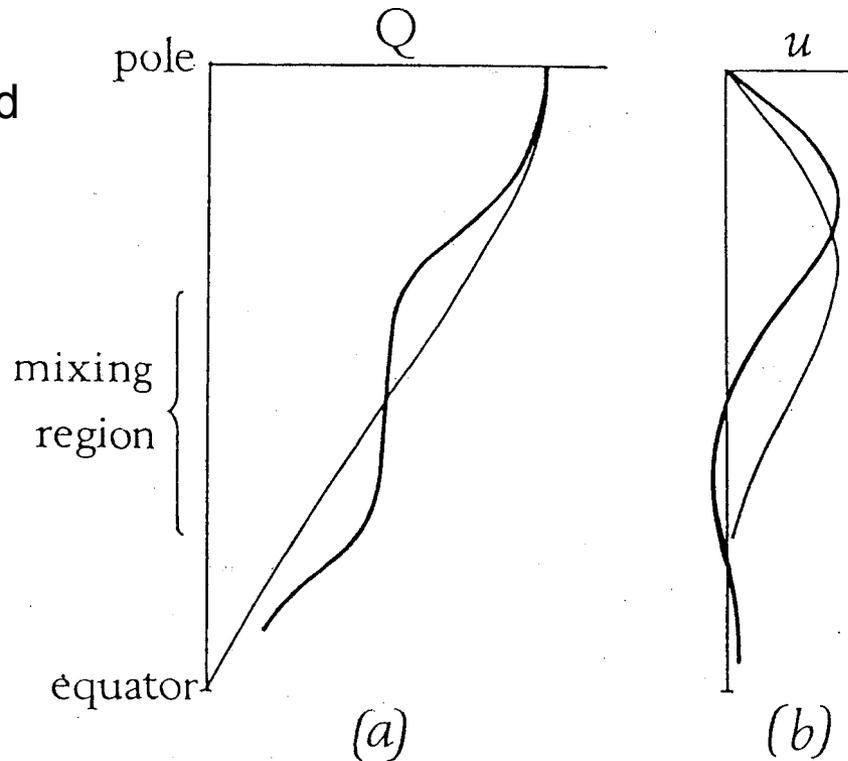
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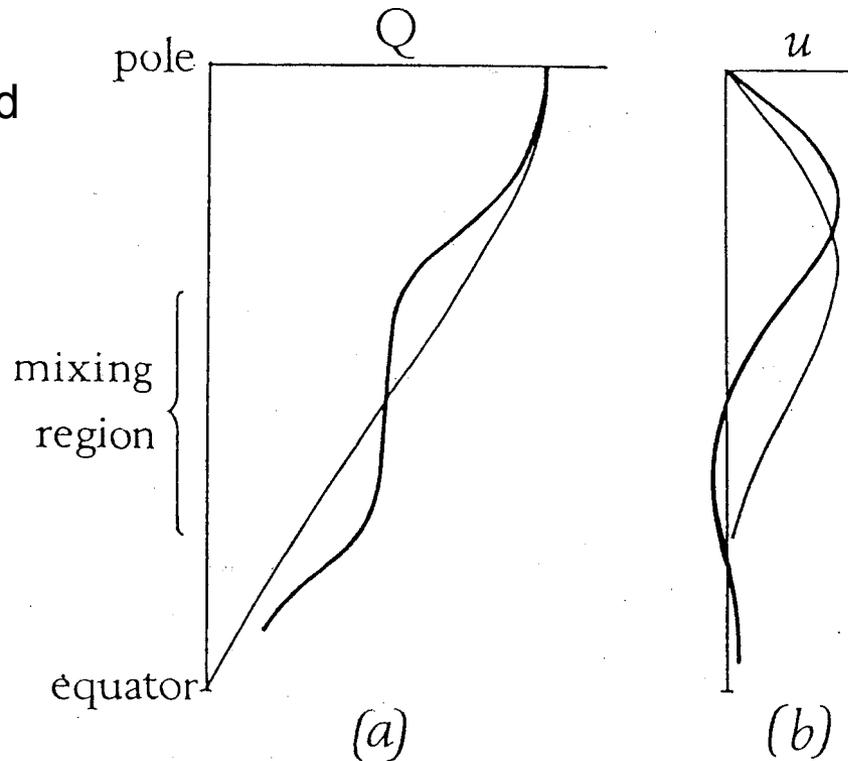


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Further insight comes from the **Taylor-Bretherton identity**. Shows how the whole jigsaw fits together, with PV mixing governing eddy momentum transport:

## The Taylor(-Bretherton) identity

(G.I. Taylor 1915, *Phil.Trans.Roy.Soc*; F. P. Bretherton 1966, *QJRMS*)

It nonlinearly relates eddy fluxes of PV to momentum-flux divergences:

Barotropic (any  $L_D$ ): 
$$\overline{v'q'} = -\frac{\partial}{\partial y} \overline{u'v'} \quad (+ \text{form stress if topog.})$$

3D baroclinic: 
$$\overline{v'q'} = \frac{1}{\rho_0} \left( \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} \right)$$

where

$$(F, G) = \rho_0(z) \left( -\overline{u'v'}, \frac{f_0 \overline{v'\theta'}}{N^2} \right) \quad (\text{“Eliassen-Palm flux”})$$
  
An arrow points from the text “form stress” to the term  $\frac{f_0 \overline{v'\theta'}}{N^2}$  in the equation above.

**NB: nonlinear relation: valid at any amplitude!** And valid regardless of whether motion is free, forced, or self-excited. Often not flagged up clearly in textbooks. **My 1982 review just as guilty** – see p. 48b.<sub>5</sub>

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PV animation from recent numerical experiments by Gavin Esler illustrating the jet self-sharpening that's so typical and ubiquitous:

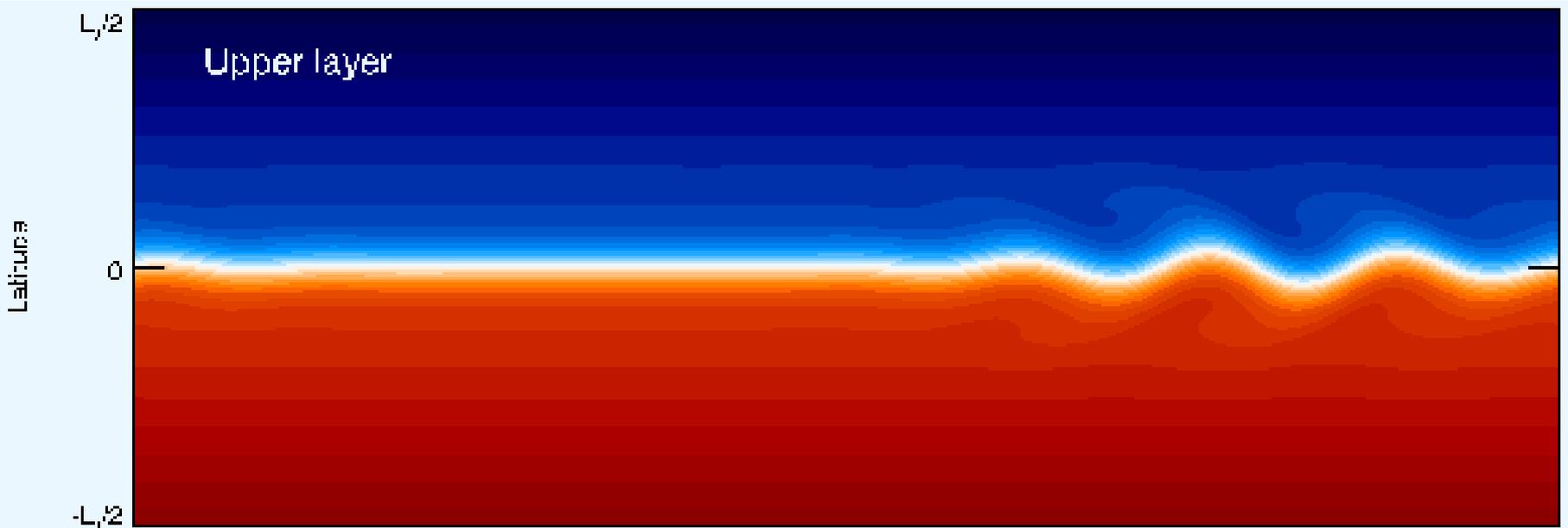
Rossby waves

(a) undulate the jet core elastically, and

(b) **break** on both sides, mixing PV and sharpening the jet's velocity profile (consequence of **PV inversion**)

The core acts as a remarkably effective "**eddy-transport barrier**" against mixing.

Esler, G., 2008, *J. Fluid Mech.* **599**, 241



Strong jets, when disturbed naturally, tend to sharpen themselves.

**A very simple toy model is insightful here:**

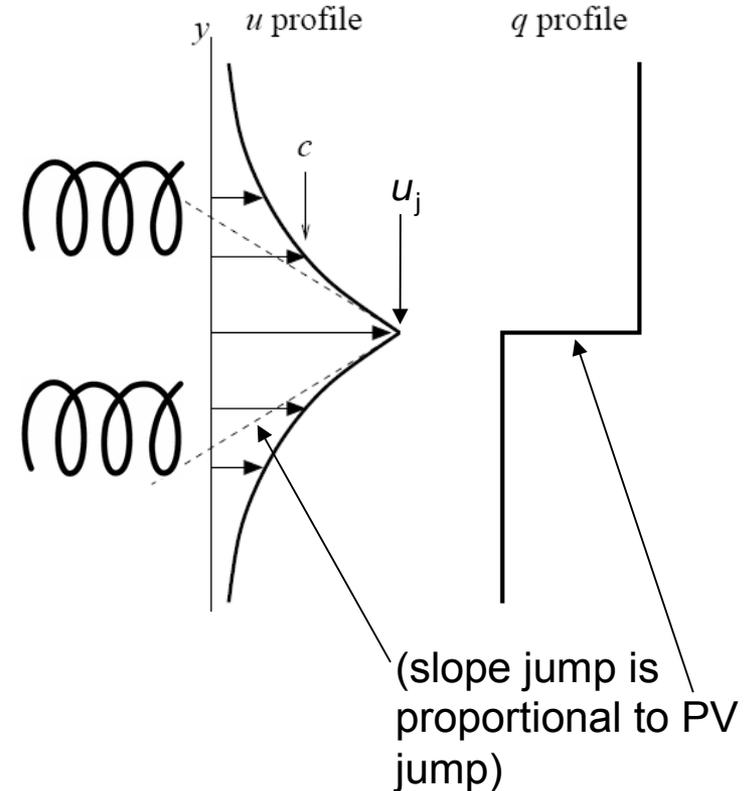
In this simplest model, the dispersion relation

$$c = u_j \left\{ 1 - (1 + L_D^2 k^2)^{-1/2} \right\}$$

implies that the phase speed  $c$  lies within the range of jet velocity profile.

So the kinematics strongly favours Rossby-wave breaking on the jet flanks. (This is the key message from nonlinear Rossby-wave critical-layer theory (Stewartson-Warn-Warn and beyond),

(The stratospheric examples are similar except that the polar-night jet self-sharpens mainly by PV mixing on its equatorward flank, as pointed out in my review.)



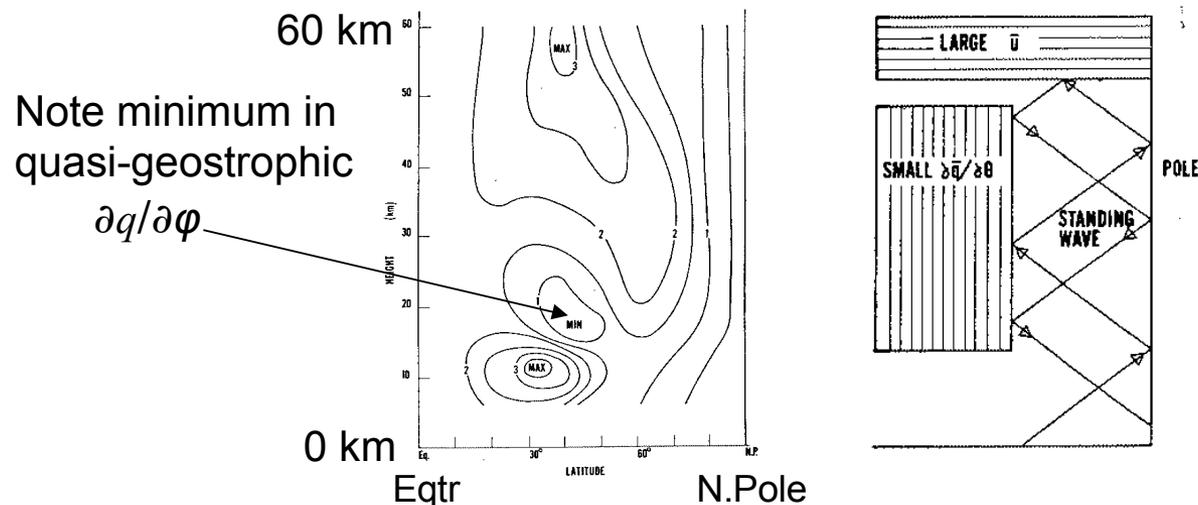
So why, then, **does the linear theory** of planetary-scale Rossby waves **do so much better than it ought?**

Two likely (and inter-related) reasons:

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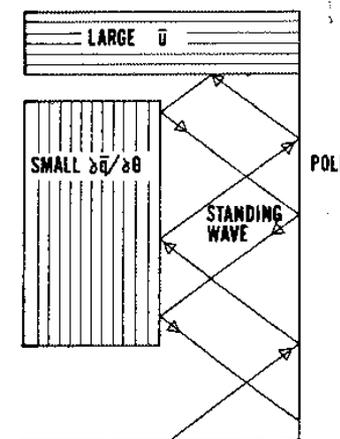
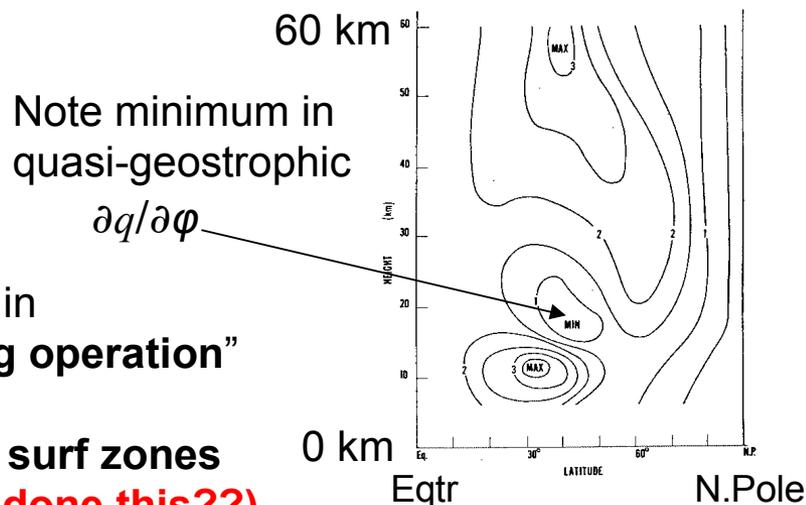
So observed PV gradients **implicitly incorporate some of the nonlinearity** through the weakness of surf-zone PV gradients; recall Matsuno 1970:

However, the refractive index **still goes crazy near critical lines.**

**Question:** sensitivity of the Chen-Robinson results to the value of the constant in a “refractive-index **trimming operation**”

$$K^2 \rightarrow \min(K^2, \text{const.})$$

– or even imposing **perfect surf zones** with  $K^2 = 0$ ? **(Has anyone done this??)**



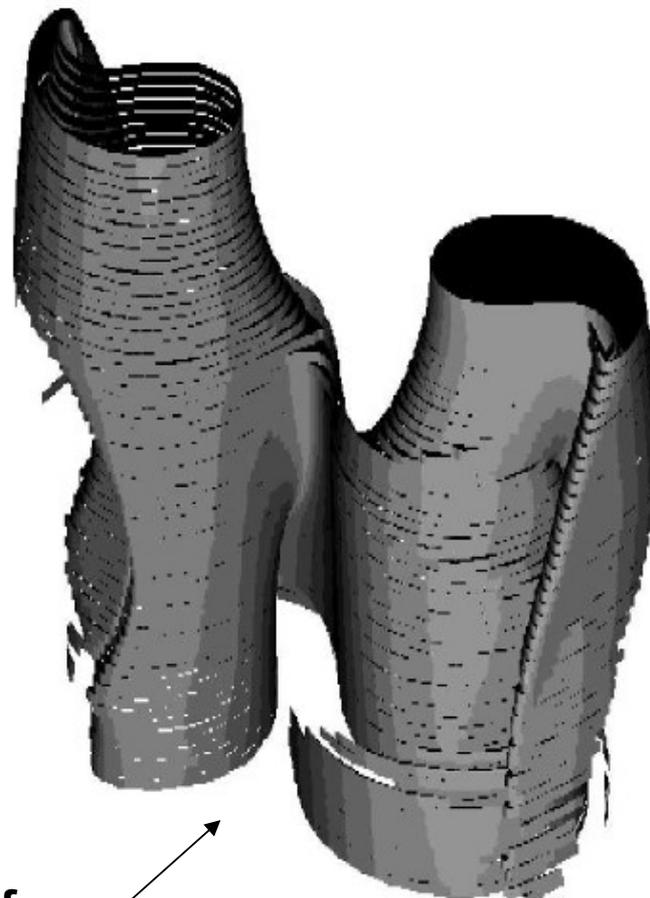
**Another question:** progress in extending idealized **perfect-surf-zone models** like that of Esler and Scott (2005, J. Atmos. Sci.) to more realistic vertical structures?

(Contour-dynamics model with perfectly-mixed surf zone, implying  $K^2 = 0$  there, hence perfect sideways reflection.)

With a realistic lower boundary condition the system has an “external” or “barotropic” mode. Even within linear theory this has its own built-in “upper reflector”.

Model has strong **self-tuning resonance by vortex erosion**, with modest forcing (quasi-topographic) – big EP uprush!

The authors judge that the **wave-2 warming of February 1979** illustrates barotropic self-tuning resonance, but not the **SH wave-2 warming of September 2002**, which had more phase tilt leading to a double helix [*sic*]. And **SH final warming** (usually wave 1)?



History reminds us how science is a struggle with unconscious assumptions.  
Here's another reminder (that we all make 'em):

Reprints, preprints & corrigenda: websearch **"lucidity principles"**  
then back to my home page at the strings "jets", "DIMBO", and "Rosenbluth"

**Wrong** unconscious assumptions I've often encountered include the following related pairs:

- { **energetics** assumption
- { **small-is-unimportant** assumption (counterexample: amplifier **input signal**)
  
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One example is where  $A = B$  is the **equation for zonal-mean temperature** or potential temperature in a model of the Brewer-Dobson circulation:  
 $B =$  **mean heating rate**,  $A =$  (TEM residual) **mean vertical velocity** times the static stability of the stable stratification.

The old idea that the right-hand side  $B$  can be regarded as **prescribed**, and the circulations as thermally **driven** is, indeed, **just an  $A = B$  assumption**.

But hang on – what's wrong with that? **Why** shouldn't I do a thought-experiment in which one prescribes the heating rate?

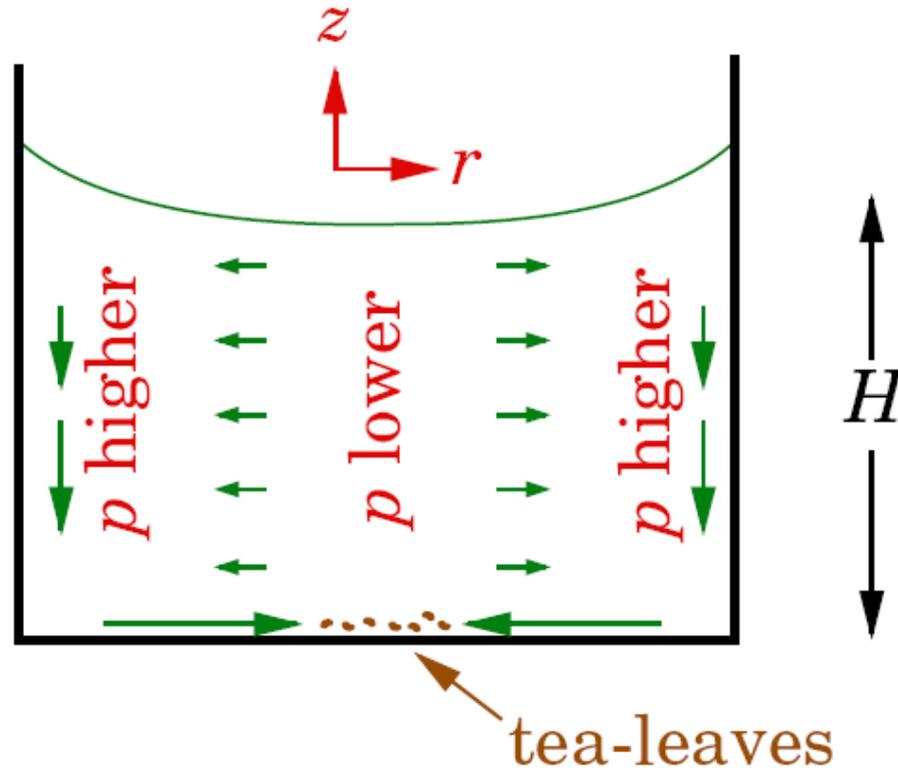
Answer: it's an **unnatural thought-experiment** in this context **because** the stratosphere and mesosphere (and the interiors of solar-type stars) are all **thermally-relaxing systems**. So it's more insightful to regard the heating rate  $B$  as part of the **response** to some forcing.

If I push a dinner-plate along a tabletop, then the friction force is part of the **response** to the force I apply. If I keep on pushing the plate, then it keeps on moving. If I stop pushing the plate, then it grinds to a halt. Its motion relaxes, frictionally, toward zero. (Would a thought-experiment **prescribing the friction force** make any sense?)

Similarly, with the stratospheric and mesospheric circulations, it's more insightful to say that they are driven not by heating but by wave-induced (**non-frictional**) **zonal forces** – mostly from breaking Rossby waves and breaking gravity waves. Keep on sending in the waves, and the circulation keeps going. It also tends to **burrow** – to extend itself **downward** from the forcing level (Haynes + 1991 *JAS*)

We may usefully describe these circulations as **gyroscopically pumped**.

**“Einstein’s Tealeaves”** demonstrates gyroscopic pumping for the special case in which the zonal force happens to be frictional:



**This experimental demonstration is very robust. It always works.**

Take a cylindrical container with a rotating mass of fluid in it. The fluid near the bottom feels a retrograde frictional force. This fluid is **gyroscopically pumped** toward the centre. The tea-leaves follow it, as in Einstein's original example of flow in a teacup.

Jet mythology, zoology, physiology, and anatomy...

The literature on jets – a complex conceptual landscape.

### **Zoology:**

1. Classic tropopause/polar-night/major-oceanic (Gulf-stream-like)
2. Mid-oceanic “striations” or “ghost jets”, e.g. Maximenko et al (2008 *GRL*)
3. Jovian jets (**straight!**)
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### **Anatomy** (2 clear extremes):

Strong jets (PV-staircase-like,  
Rossby waves **guided**)

Weak jets (PV close to large-scale  
background beta, Rossby waves  
**unguided**, quasi-plane)

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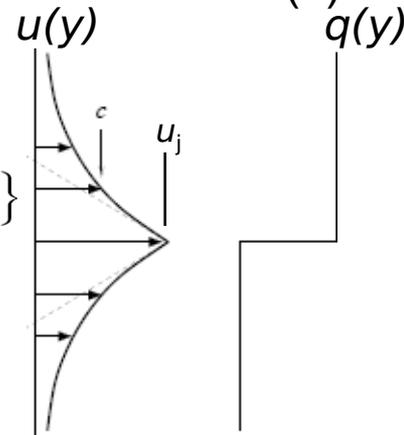
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Strong jets (PV-staircase-like, Rossby waves **guided**)

Weak jets (PV close to large-scale background beta, Rossby waves **unguided**, quasi-plane)

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(Rosenbluth Lecture):

$$c = u_j \{1 - (1 + L_D^2 k^2)^{-1/2}\}$$



The literature on jets – a complex conceptual landscape.

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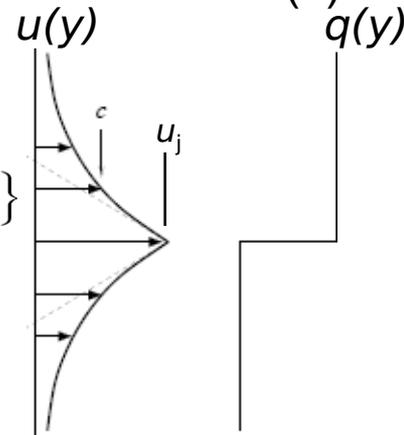
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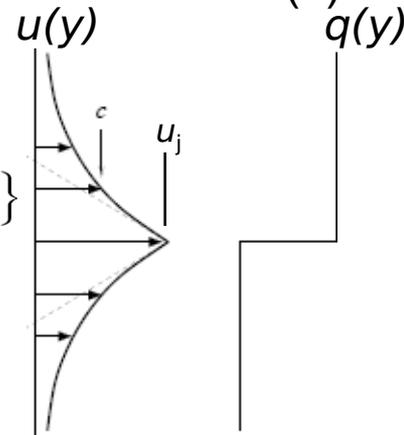
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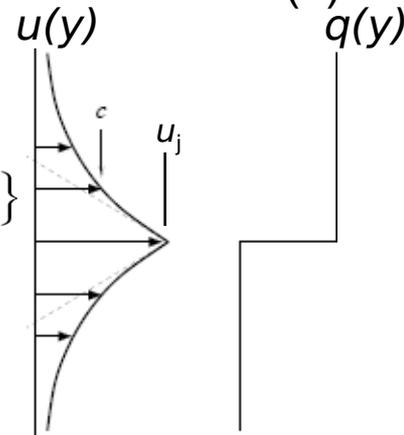
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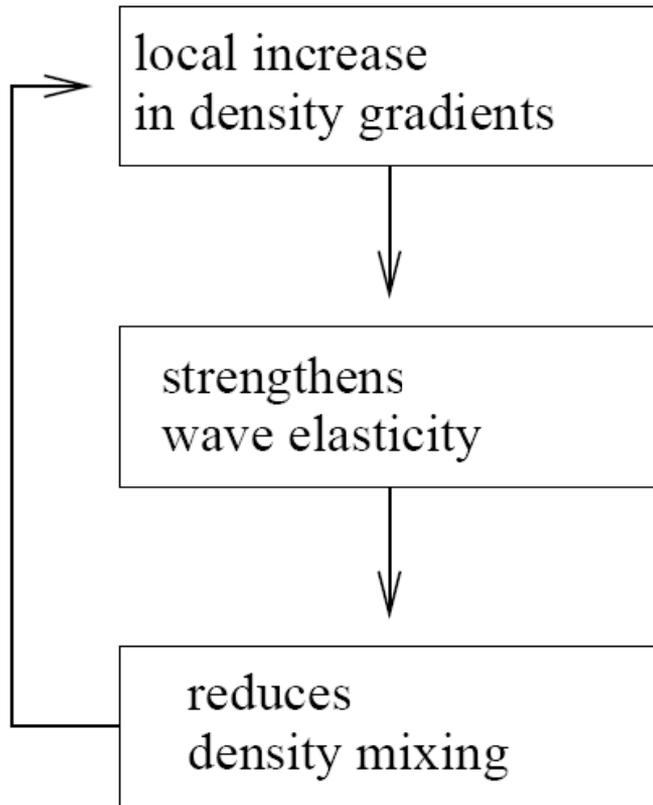
$\rightarrow$  Rhines scale



**Not so clear:** hyper-strong, hyper-staircase-like? **Jupiter?** (Dowling 1993, *JAS*)

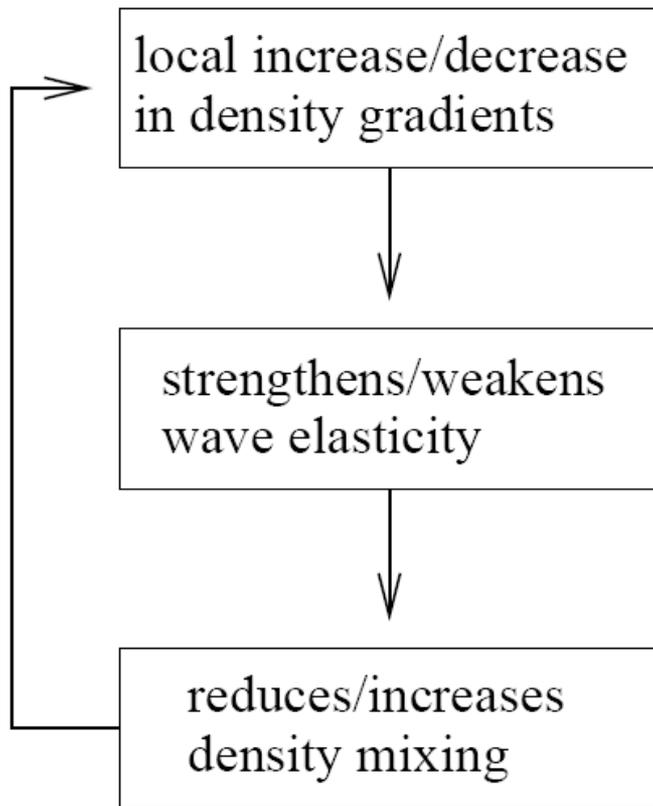
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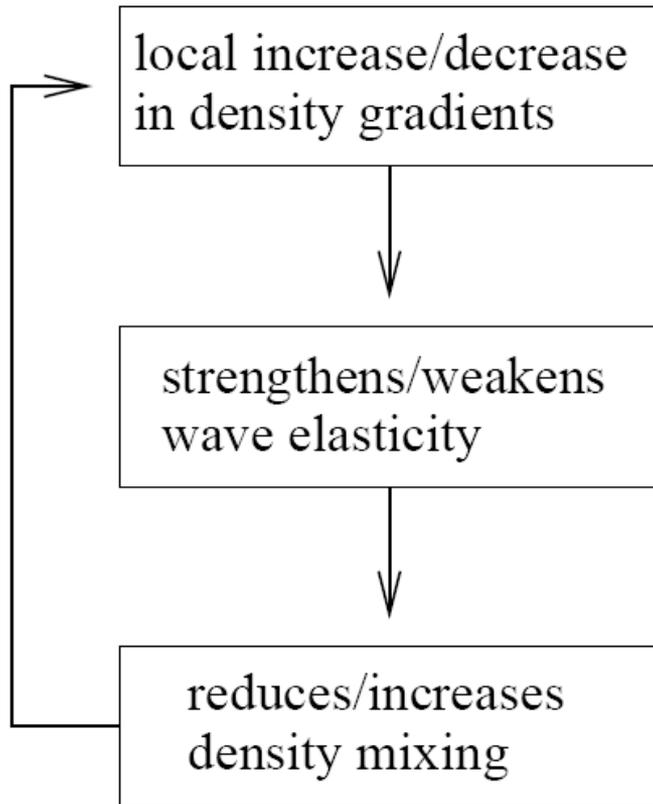
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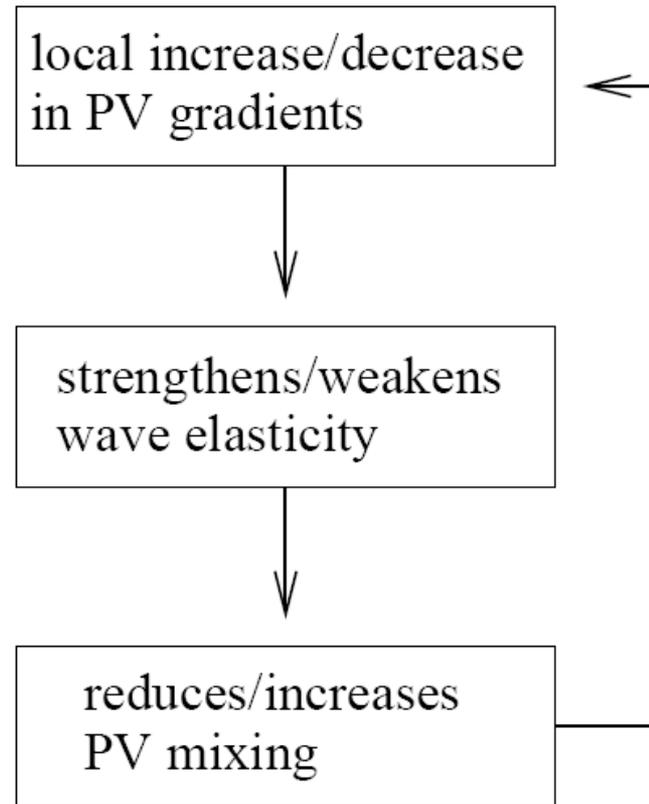


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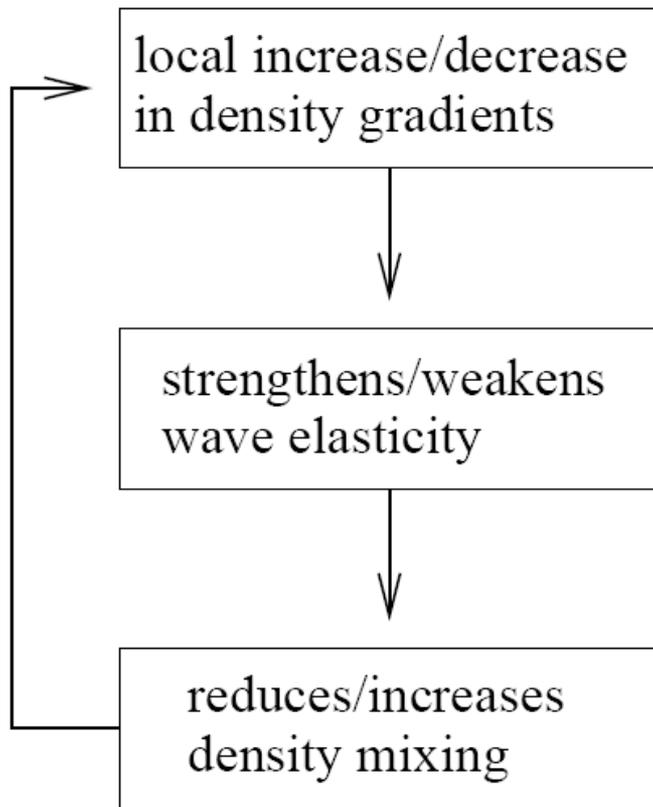


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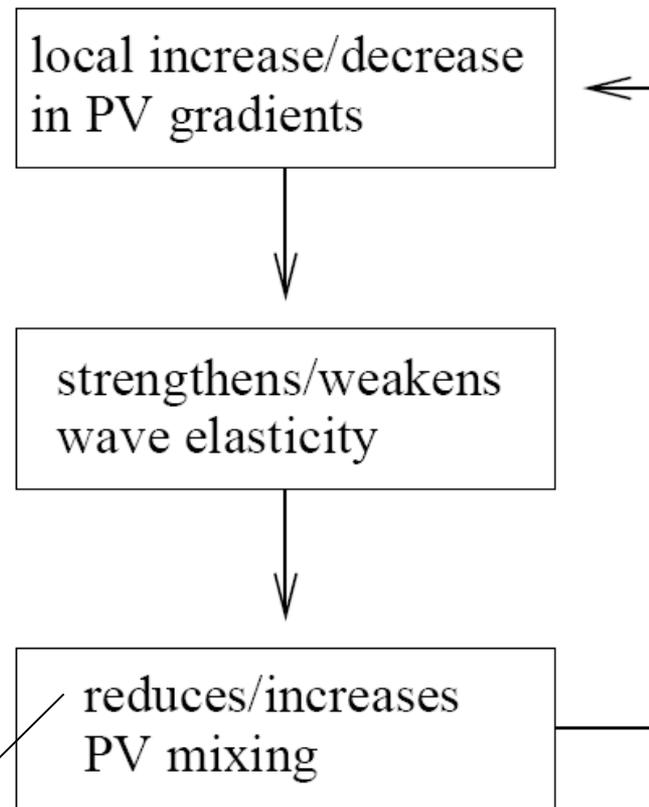


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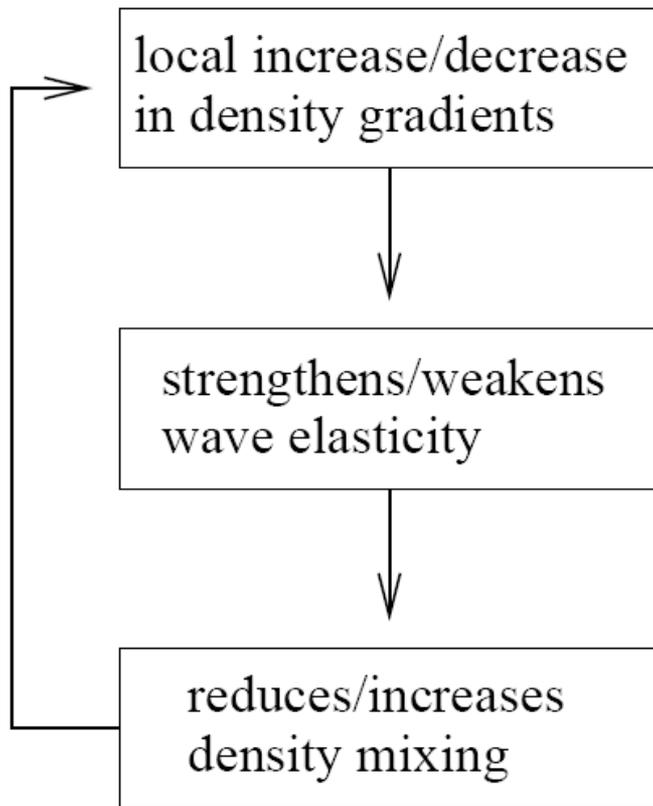
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(PV inversion then gives jets.)

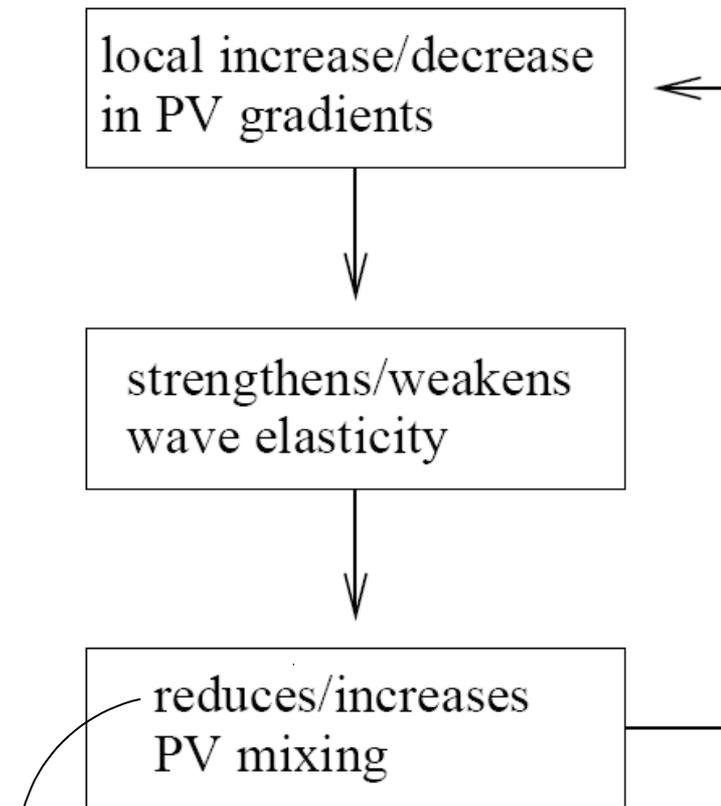
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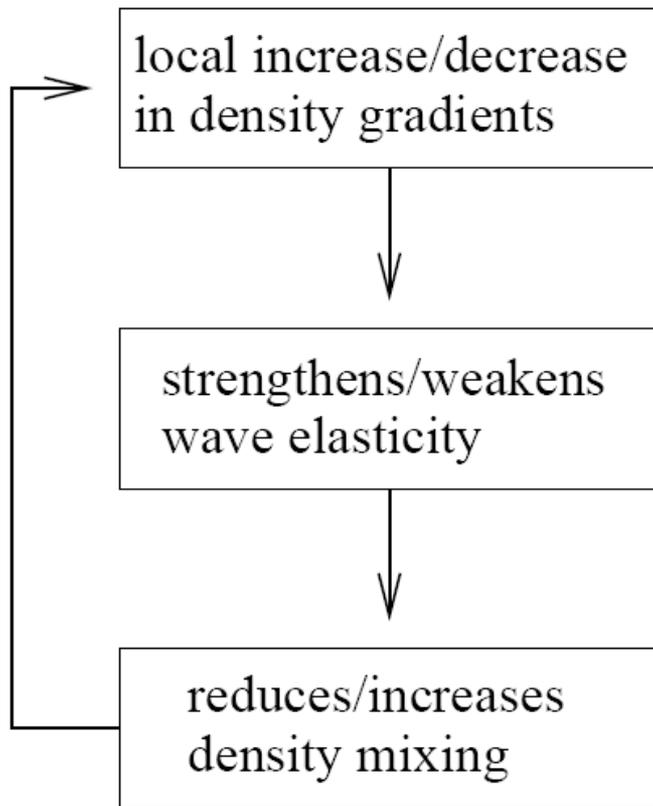
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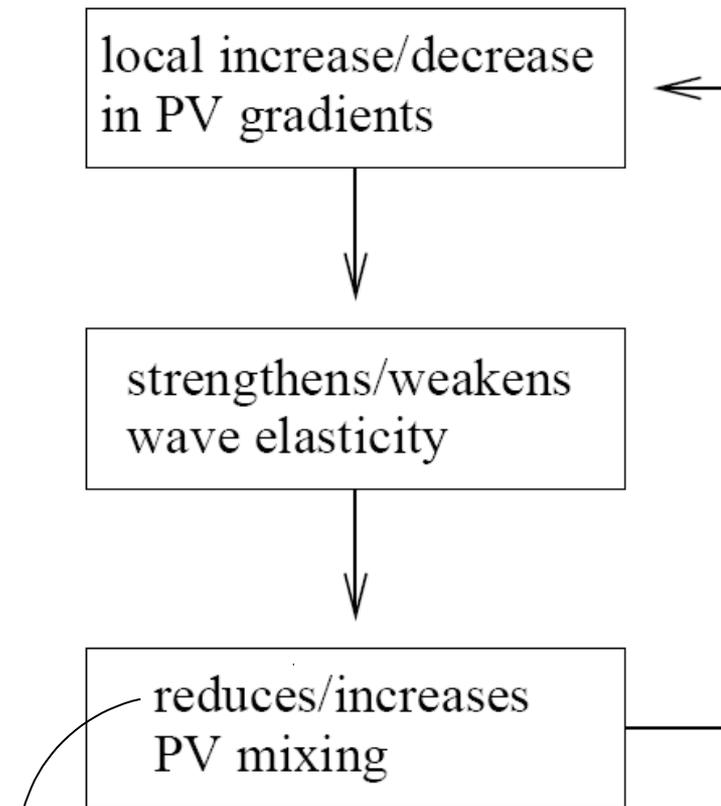
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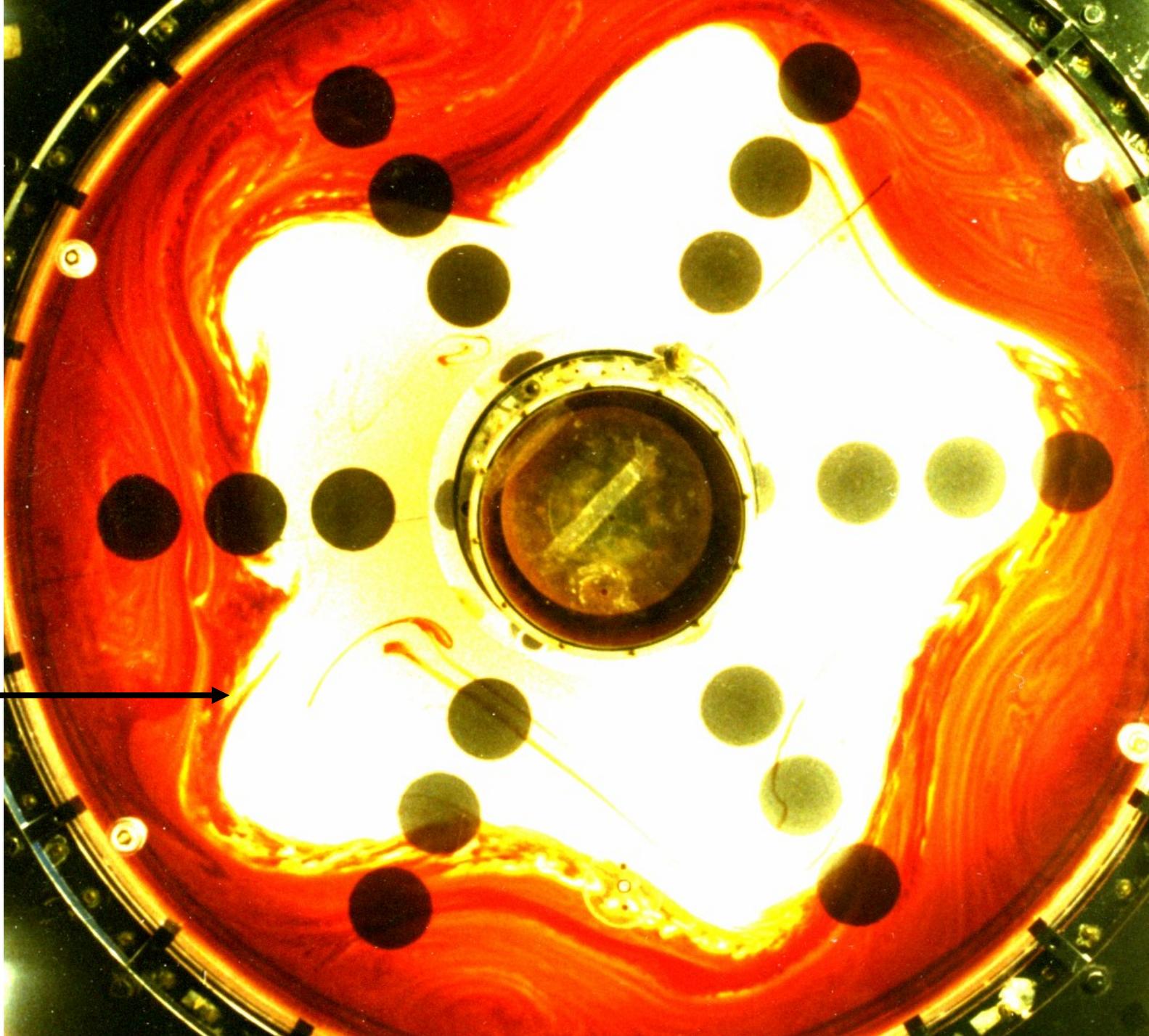
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**Here's a classic lab. demo. of a strong jet:**

Sommeria,  
Myers, and  
Swinney,  
*Nature* 1989  
86.4 cm dia.;  
rotation  $\sim$   
20 rad/s (!)

**PV map** and  
**dye map**  
near-identical.

This is clearly  
a **strong jet**:  
staircase-like;  
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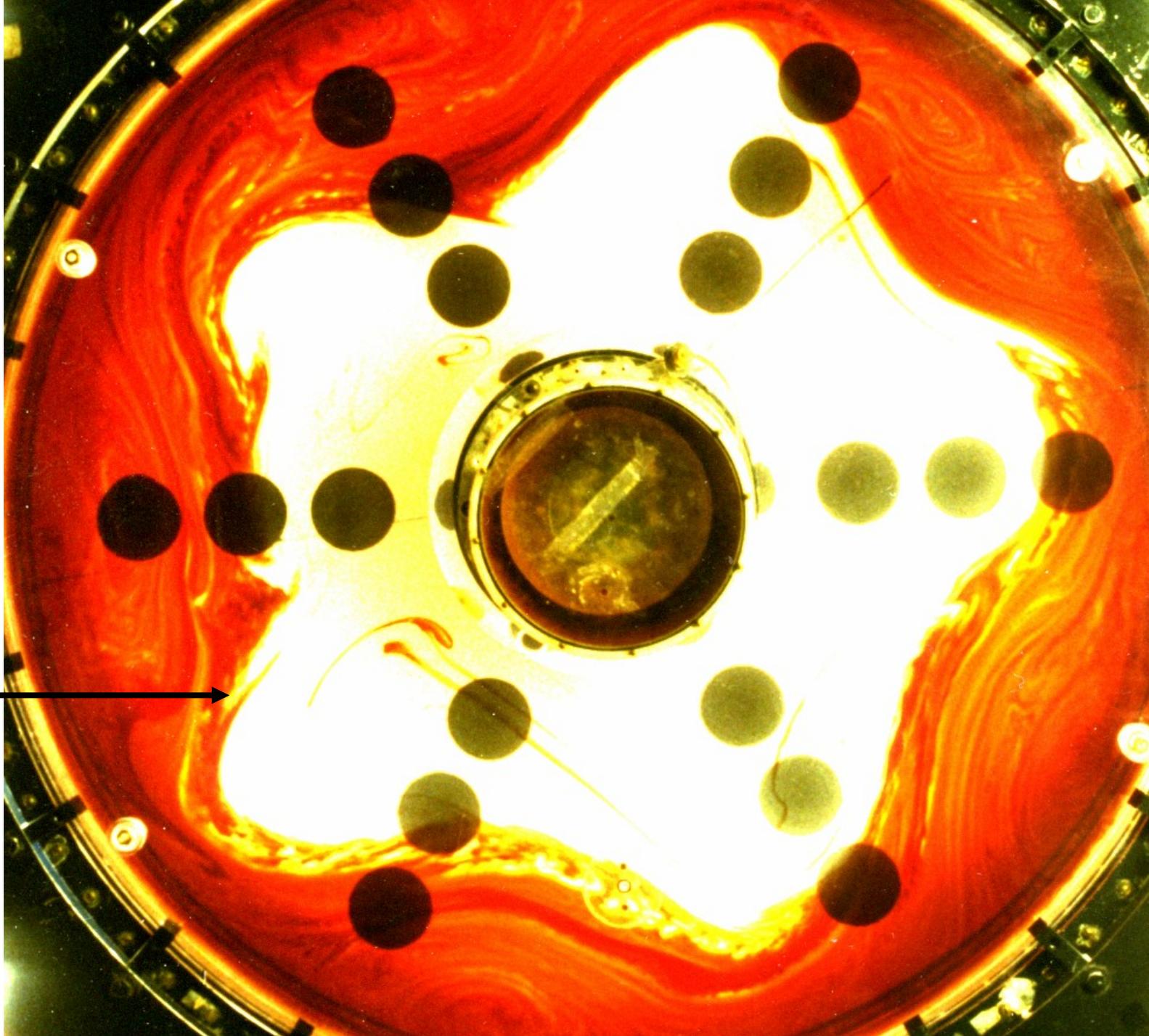


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By the way:  
**no upscale  
cascade**  
is involved.  
(Surprise??)



Model stratospheres are similar  
(Jukes & M 1987):

Polar-night jet strengthened and  
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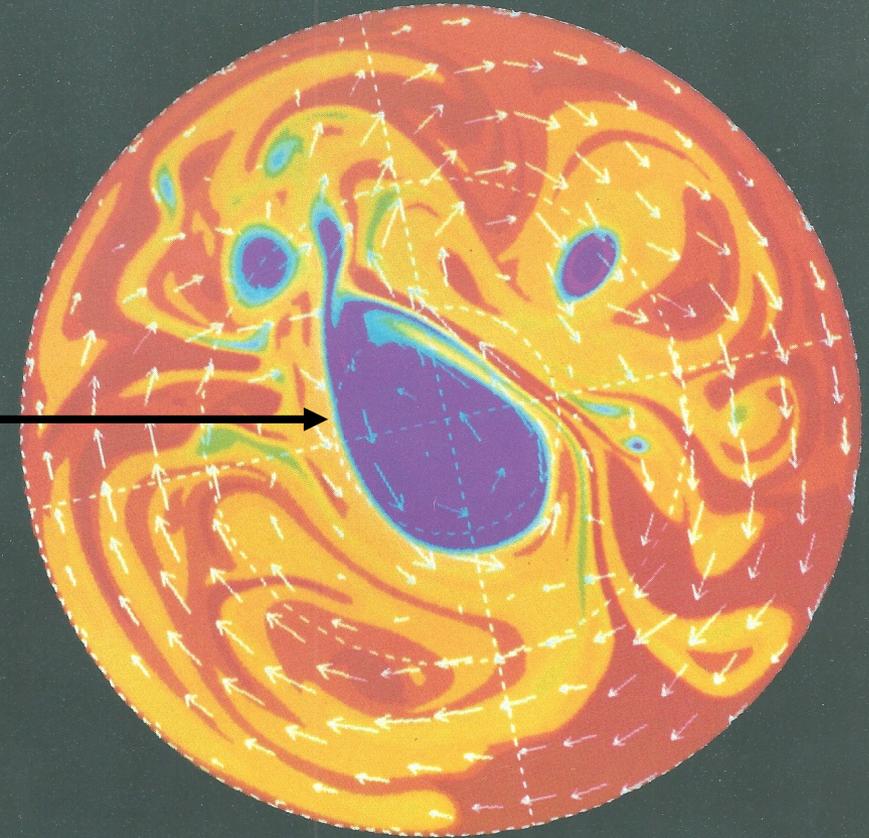
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# nature

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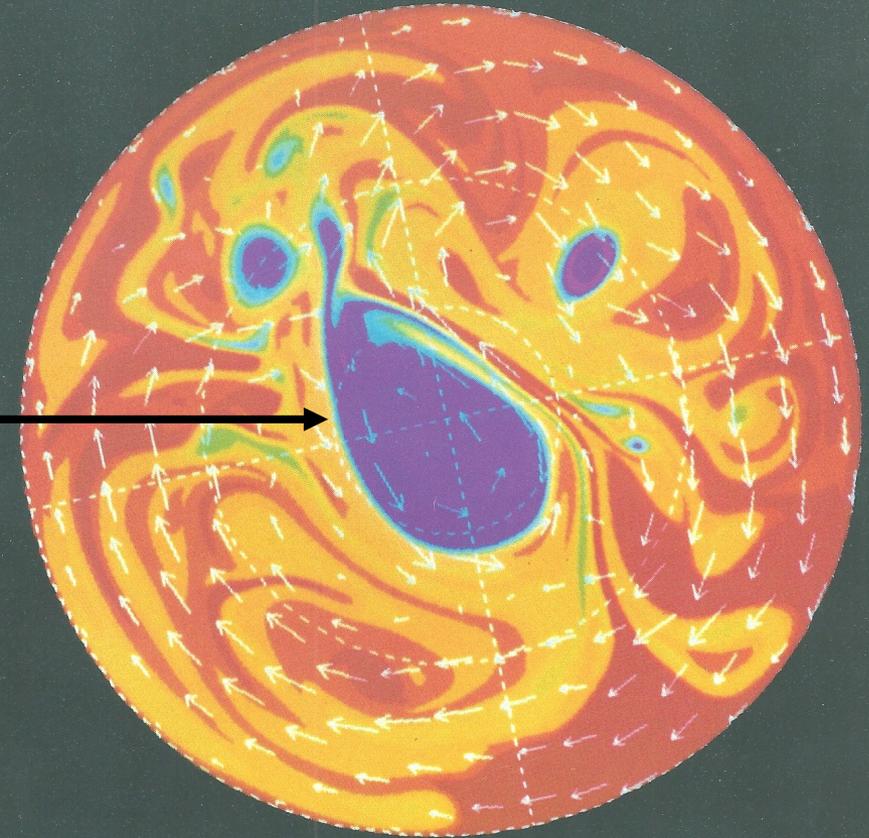
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Same for the **real** stratosphere.

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Summary: 2-level hierarchy of ideas for understanding the fluid dynamics of jets

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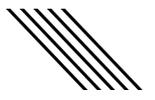
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Taylor-Bretherton identity  
 $\overline{v'q'} = -\text{div}(\text{eddy momentum flux})$   
**Nonlinear, forced/free/self-excited**

2. **Particular mechanisms:**

i) Rhines effect. Re **weak** jets generated by strong small-scale forcing – strong enough to create **active** small-scale vortices that merge or cluster, producing an **upscale cascade** that is arrested or slowed when eddy velocities  $\sim$  **plane** Rossby-wave phase speeds. Wave-turbulence interaction is spatially **homogeneous**.

(ii) Jet self-sharpening by Rossby-wave breaking. Re jets **strong** enough to be Rossby waveguides. Wave-turbulence interaction spatially **inhomogeneous**.

(iii) Repeated excitation of **Kelvin sheared disturbances** by small-scale forcing weaker than in (i). (Kelvin 1887, Farrell and Ioannou 2007 & refs.). 

(iv) Downstream wind stress reinforcing strong ocean jets (e.g. Thomas & Lee'05 *JPO*)

Reprints, preprints & corrigenda: websearch "**lucidity principles**" then back to my home page at the strings "jets", "DIMBO", and "Rosenbluth"

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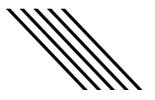
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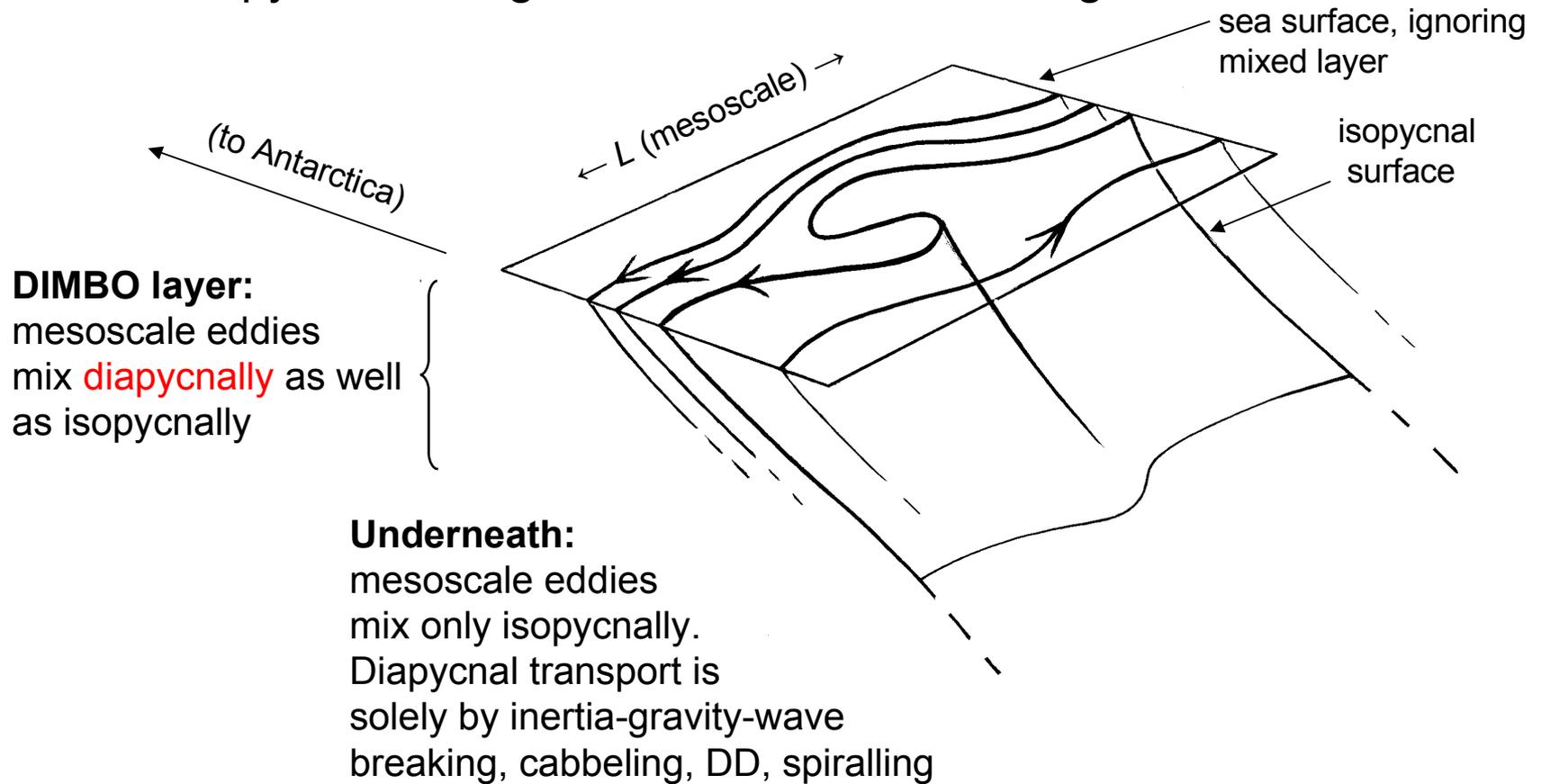
**MYTH: “Mechanism (i) is universal.”**

**Stratosphere disproves this: clearly (ii). For Jupiter I'm betting on (iii) .**

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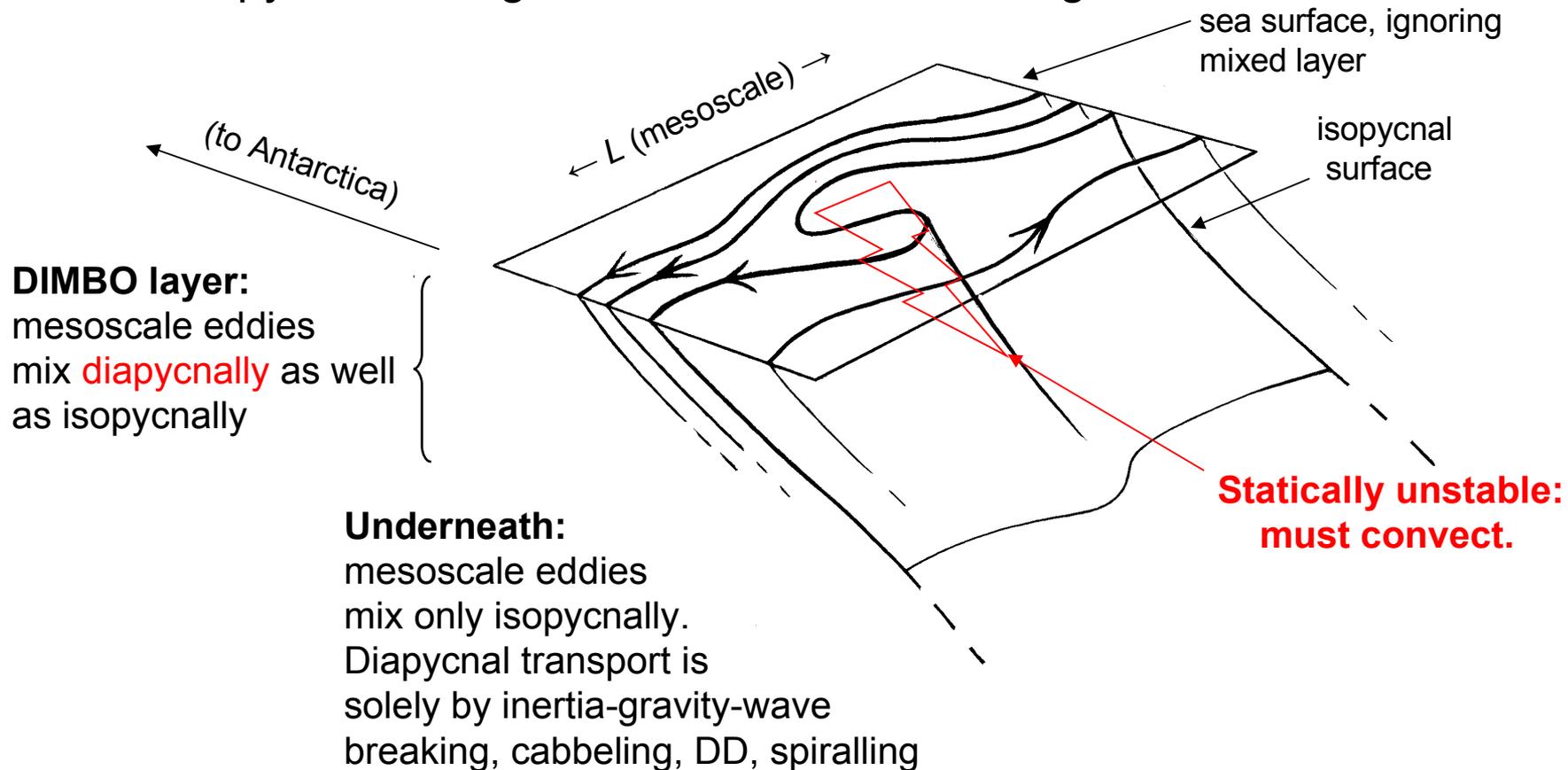
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**How deep is the DIMBO layer?** Scale analysis and semigeostrophic PV inversion suggest the “obvious” answer  $fL/N$ . Could ~ kilometre or two. Must often exceed mixed-layer depth.

Numerical experiments underway (John Taylor, Raff Ferrari, personal communication)  
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The old idea that the right-hand side  $B$  can be regarded as **prescribed**, and the circulations as thermally **driven** is, indeed, **just an  $A = B$  assumption**.

But hang on – what’s wrong with that? What’s wrong with a thought-experiment in which one prescribes the heating rate?

Answer: it’s an unnatural thought-experiment in this context **because** the stratosphere and mesosphere – and the interiors of solar-type stars – are all **thermally-relaxing systems**. So it’s more insightful to regard the heating rate  $B$  as part of the **response** to some forcing.

If I push a dinner-plate along a tabletop, then the friction force is part of the **response** to the force I apply. If I keep on pushing the plate, then it keeps on moving. If I stop pushing the plate, then it grinds to a halt. Its motion relaxes, frictionally, toward zero. (Would a thought-experiment **prescribing the friction force** make any sense?)

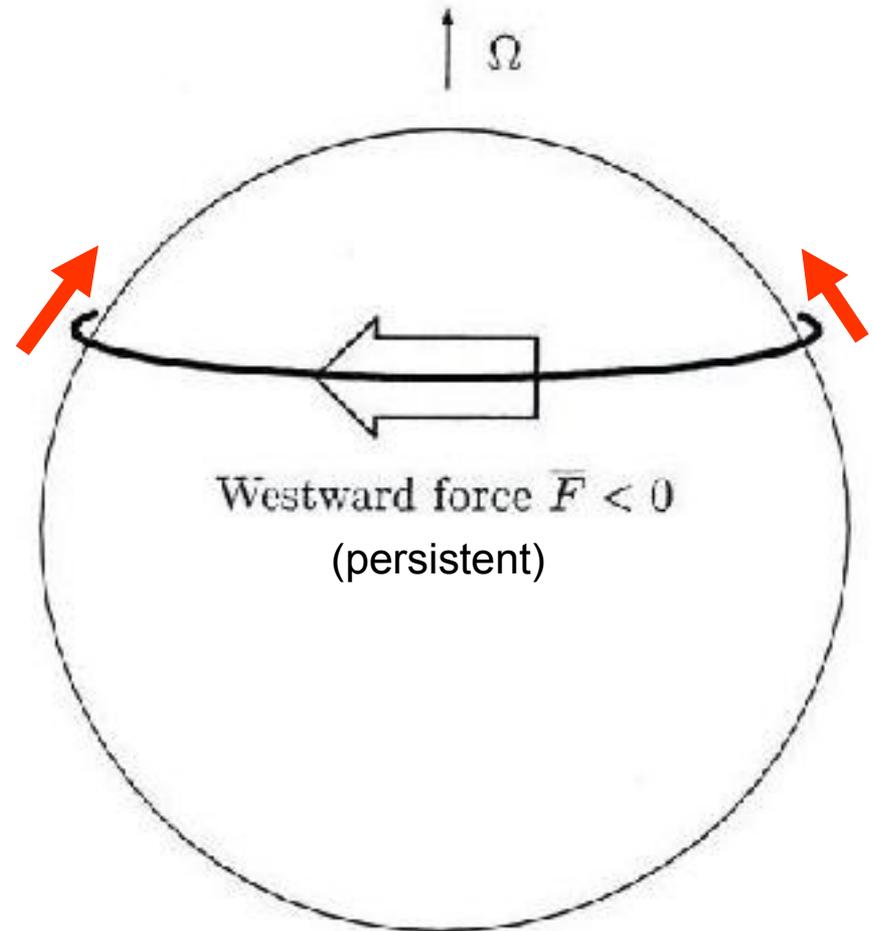
Similarly, with the stratospheric and mesospheric circulations, it’s more insightful to say that they are driven not by heating but by wave-induced (**non-frictional**) **zonal forces** – mostly from breaking Rossby waves and breaking gravity waves. Keep on sending in the waves, and the circulation keeps going. It also tends to **burrow** – to extend itself **downward** from the forcing level (Haynes + 1991 *JAS*)

We may usefully describe these circulations as **gyroscopically pumped**.

# Gyroscopic pumping is **easy** to understand:

Rapidly-rotating system!  
Low Rossby number,  
Coriolis effects are **strong**.

Coriolis force turns  
fluid poleward: a robust  
and systematic **mechanical  
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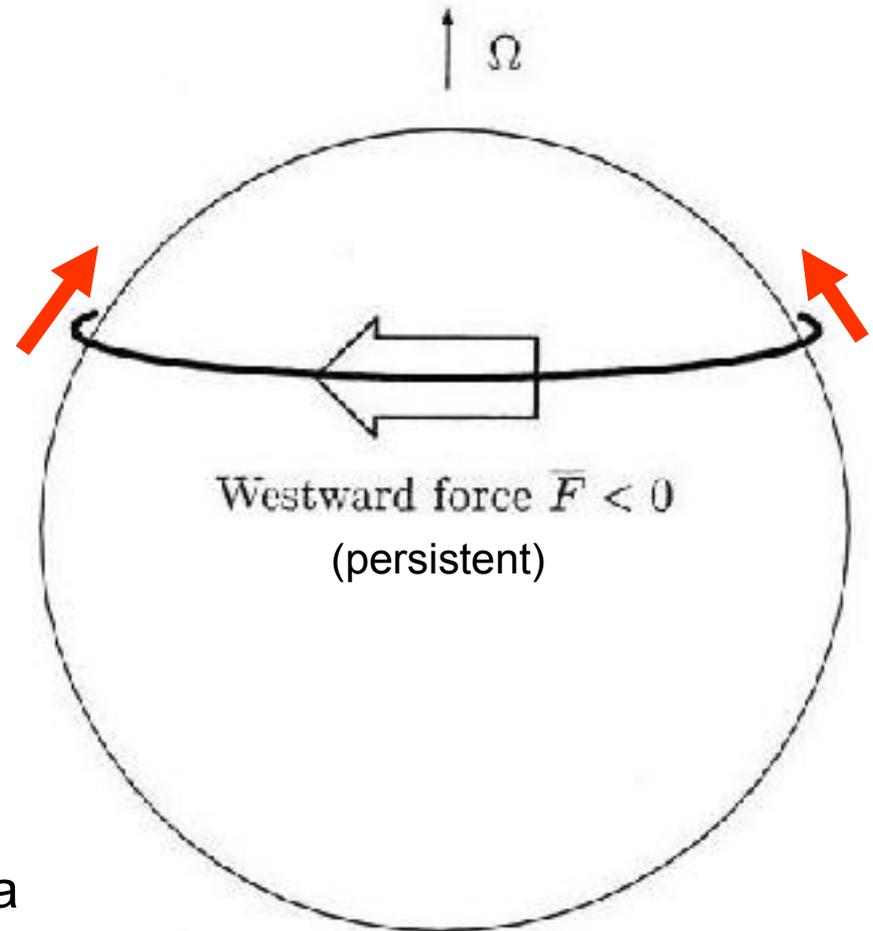


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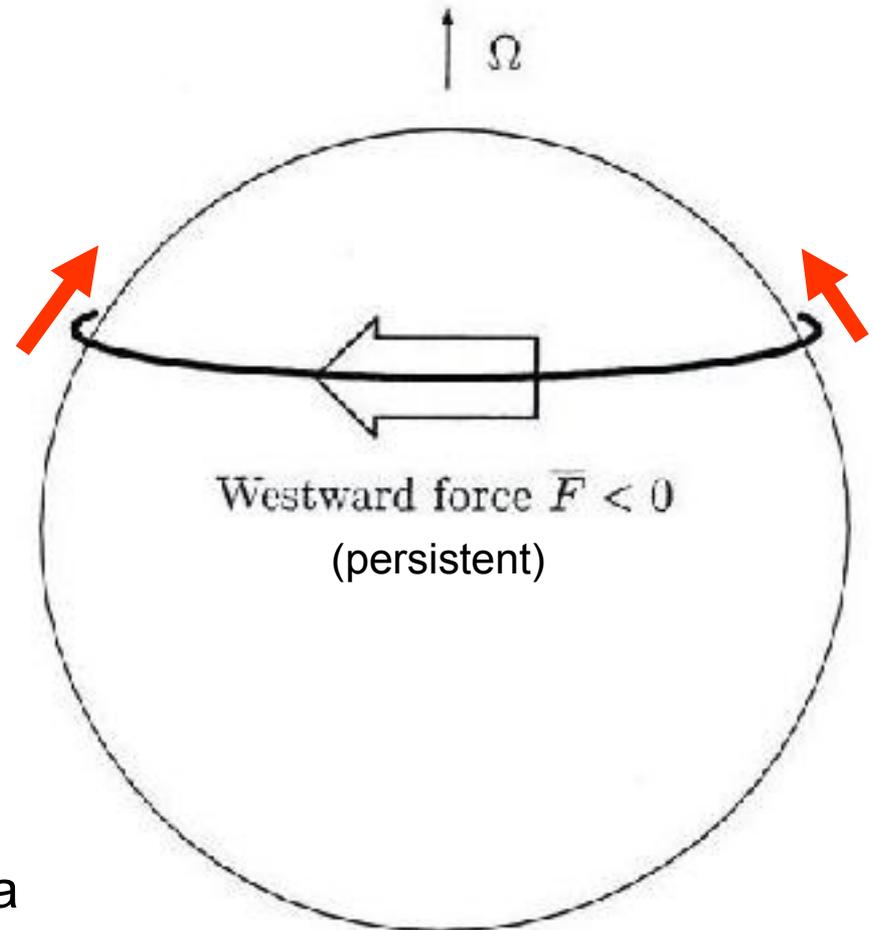
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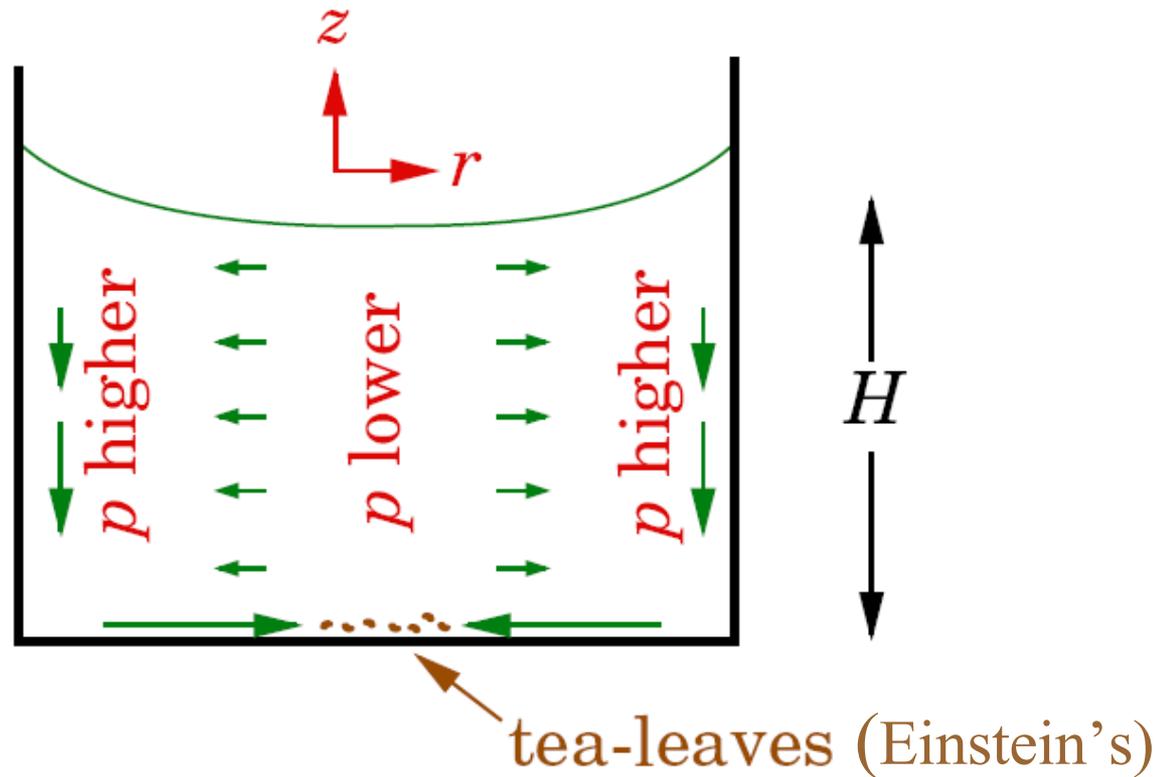
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**Answer: "downward**

**control"**

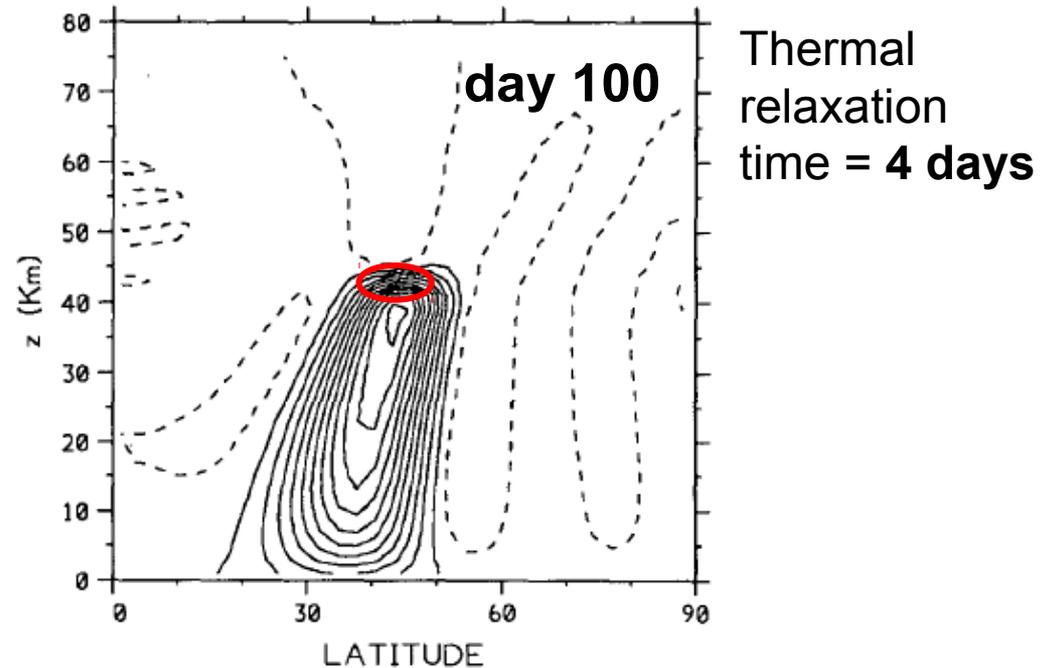
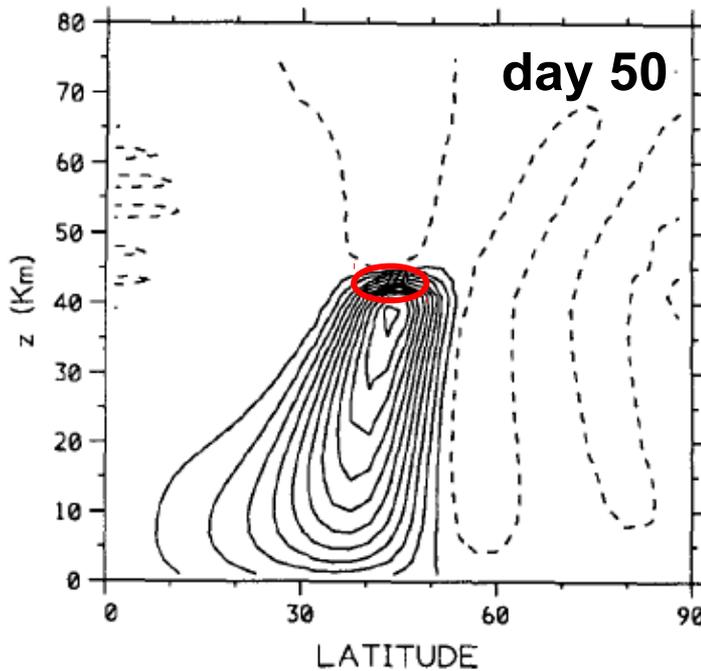
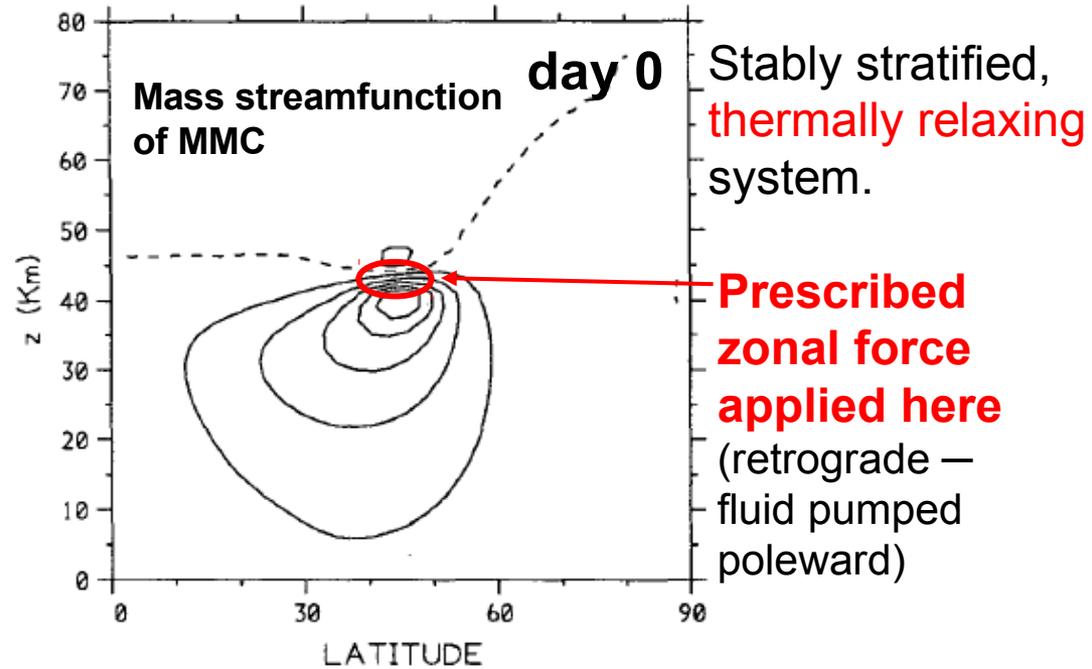
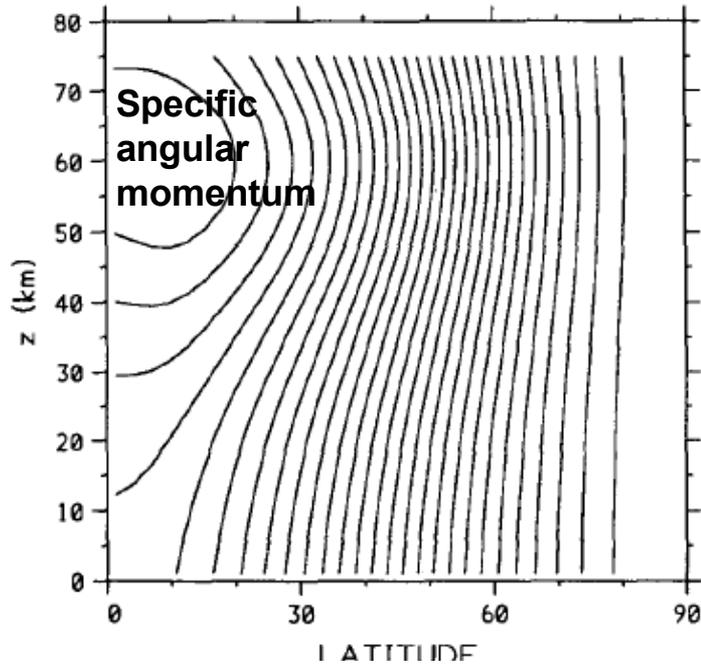


**This experimental demonstration is very robust. It always works.**

Take a cylindrical container with a rotating mass of fluid in it. The fluid near the bottom feels a retrograde frictional force. This fluid is gyroscopically pumped toward the centre. The tea-leaves follow it, as in Einstein's original example of flow in a teacup.

**But how does a thermally relaxing, stably-stratified system respond to gyroscopic pumping at some altitude? Ans: “downward control”**

# Terrestrial cases with a lower boundary (Haynes, P. H., et al., 1991):



Mass streamfunction of MMC in the final steady state is given by a simple formula that we call the **“downward-control integral”**:

**Prescribed zonal force**

$$\psi(\phi, z) = \int_z^\infty \left\{ \frac{\rho_0 a^2 \bar{\mathcal{F}} \cos^2 \phi}{\bar{m}_\phi} \right\}_{\phi=\phi(z')} dz'$$

latitude

integral is along a characteristic of the backg. ang. mtm  $\bar{m}$

Immediate consequence of mass conservation together with the **“gyroscopic-pumping relation”**

$$\frac{\partial \psi}{\partial \phi} \frac{\partial \bar{m}}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial \bar{m}}{\partial \phi} = \rho_0 a^2 \bar{\mathcal{F}} \cos^2 \phi$$

(hyperbolic operator!)

Haynes, P. H., et al., 1991: J. Atmos. Sci., **48**, 651-678

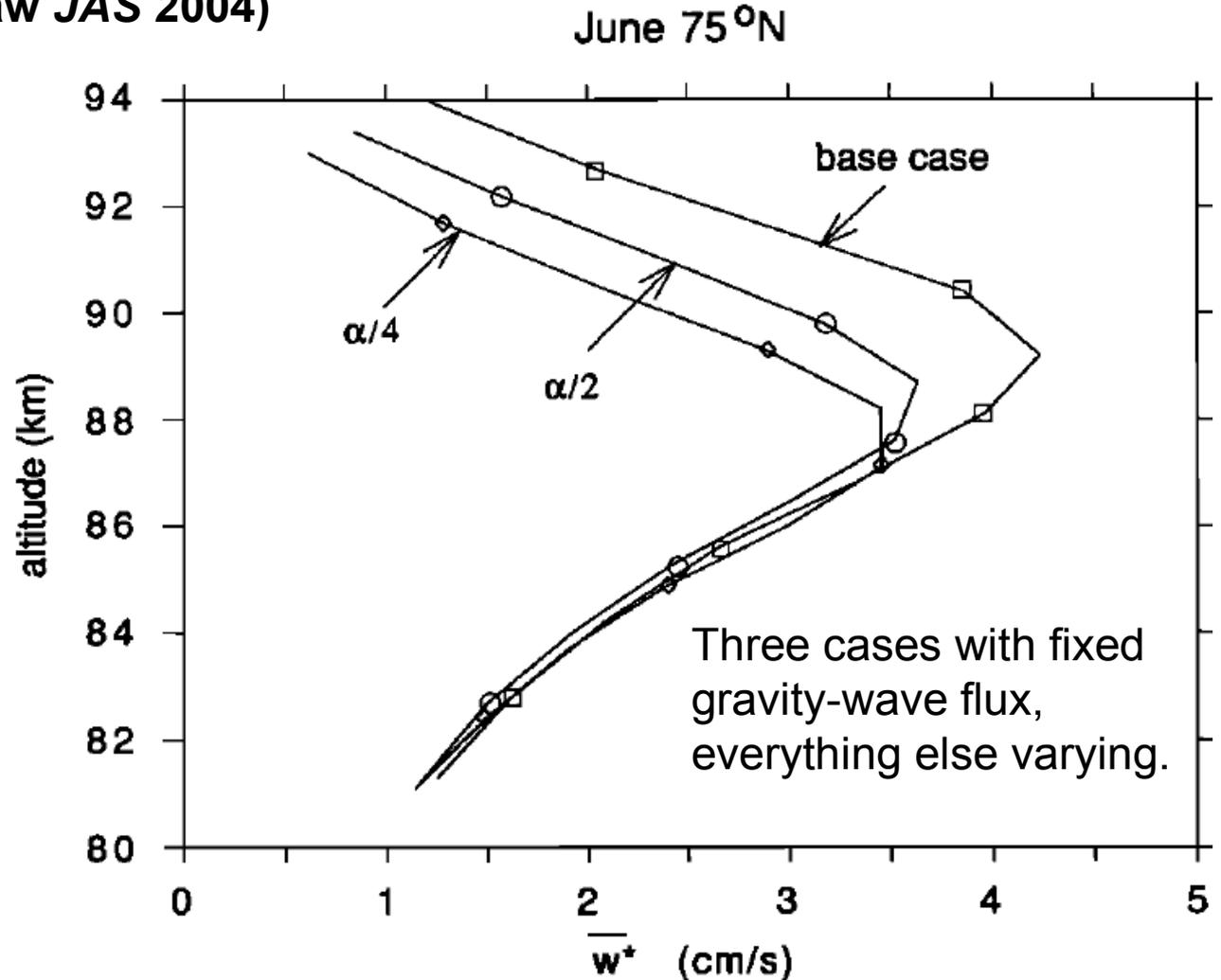
**When force is due entirely to upward-propagating gravity waves, and Rossby number small, there is an interesting simplification, relevant to the cold summer mesopause:**

E.g., **polar mesospheric upwelling** depends only on gravity-wave flux from below:

$$\bar{w}_0^*(y, z) = -\frac{1}{\cos \varphi} \frac{\partial}{\partial y} \{f^{-1}(y) \cos \varphi \overline{u'w'}\}$$

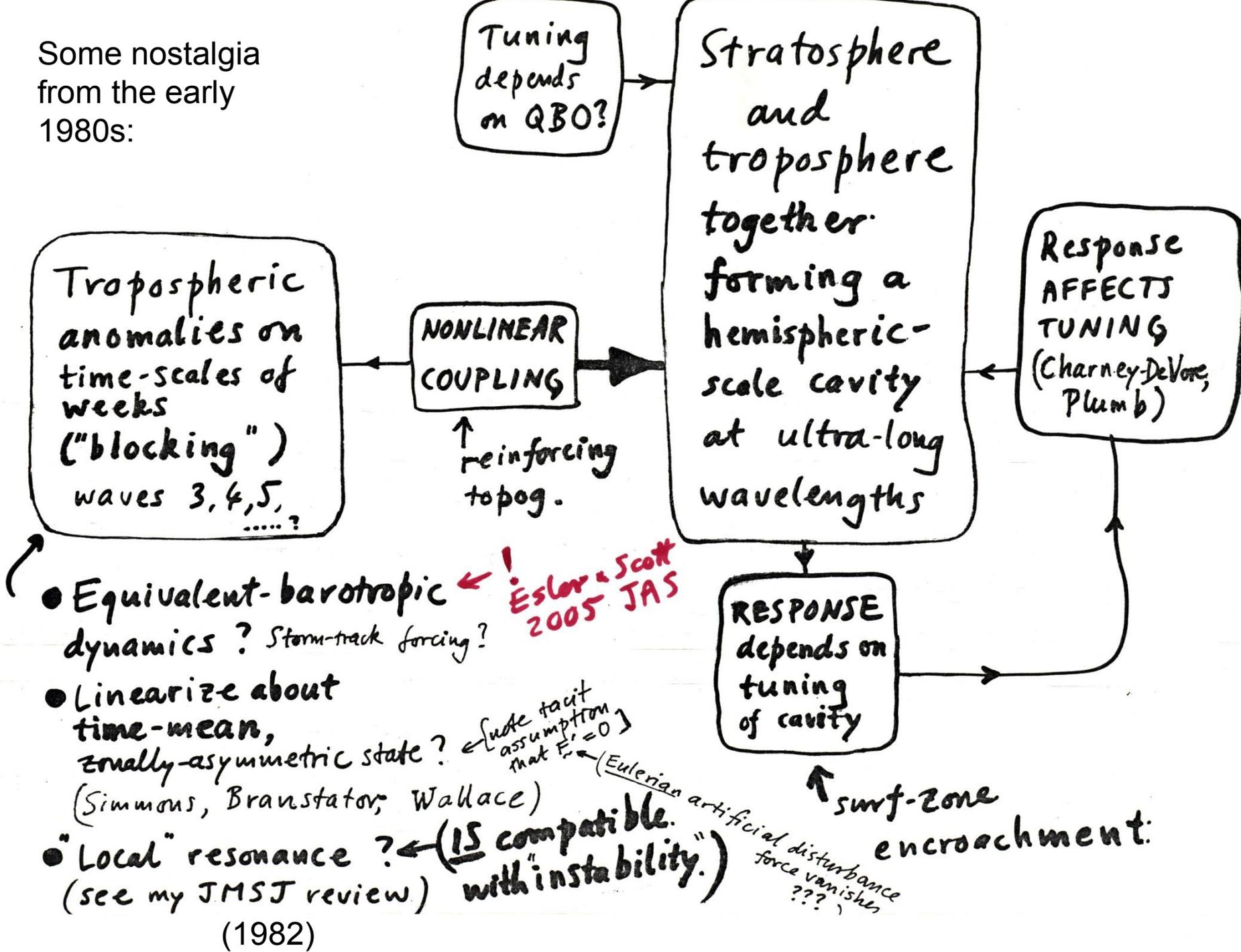
(McIntyre, *JGR* 1989, special issue on noctilucent clouds; also Shepherd and Shaw *JAS* 2004)

Waves  
break above  
Garcia model  
mesopause  
(same *JGR* issue)





Some nostalgia from the early 1980s:

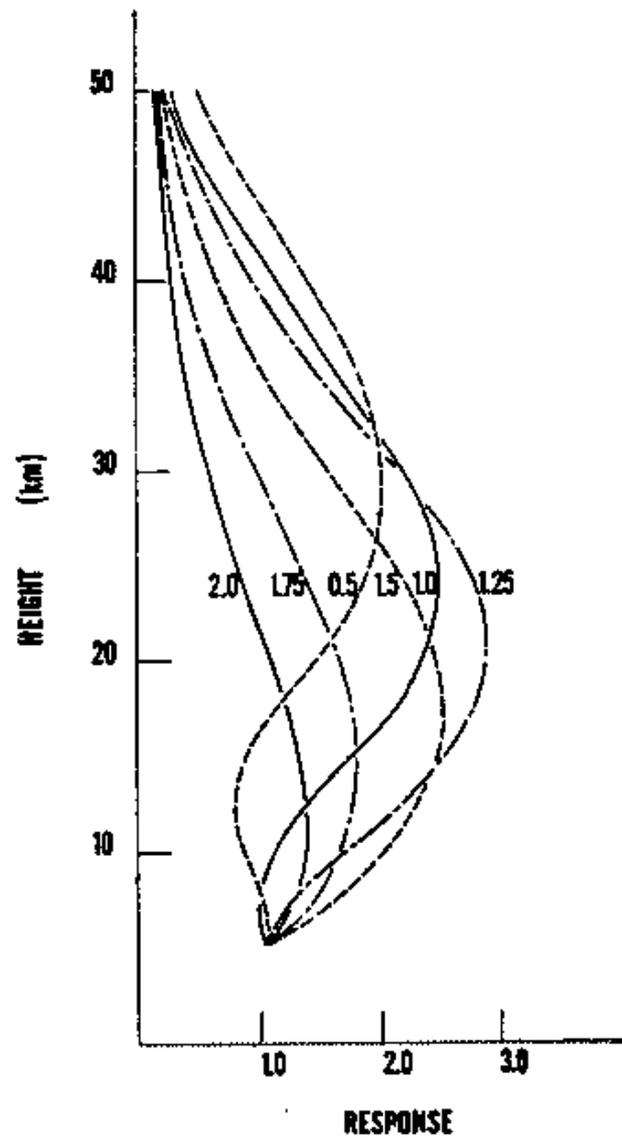
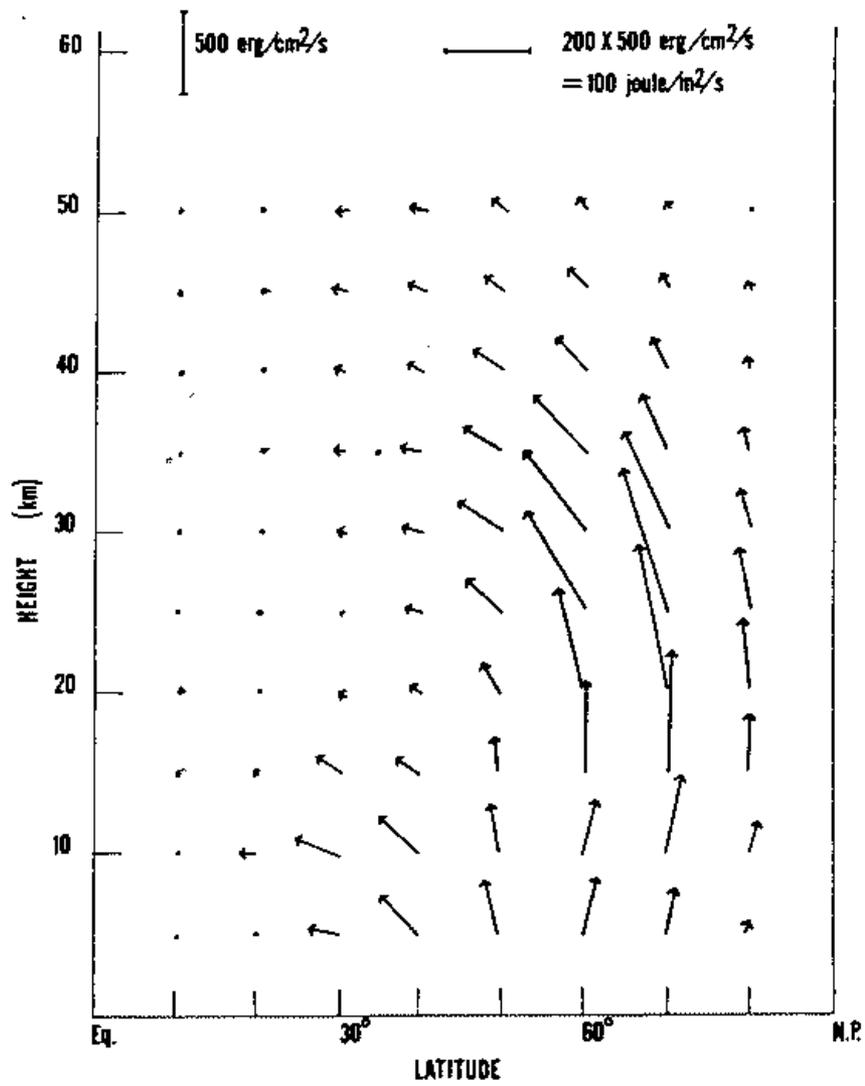


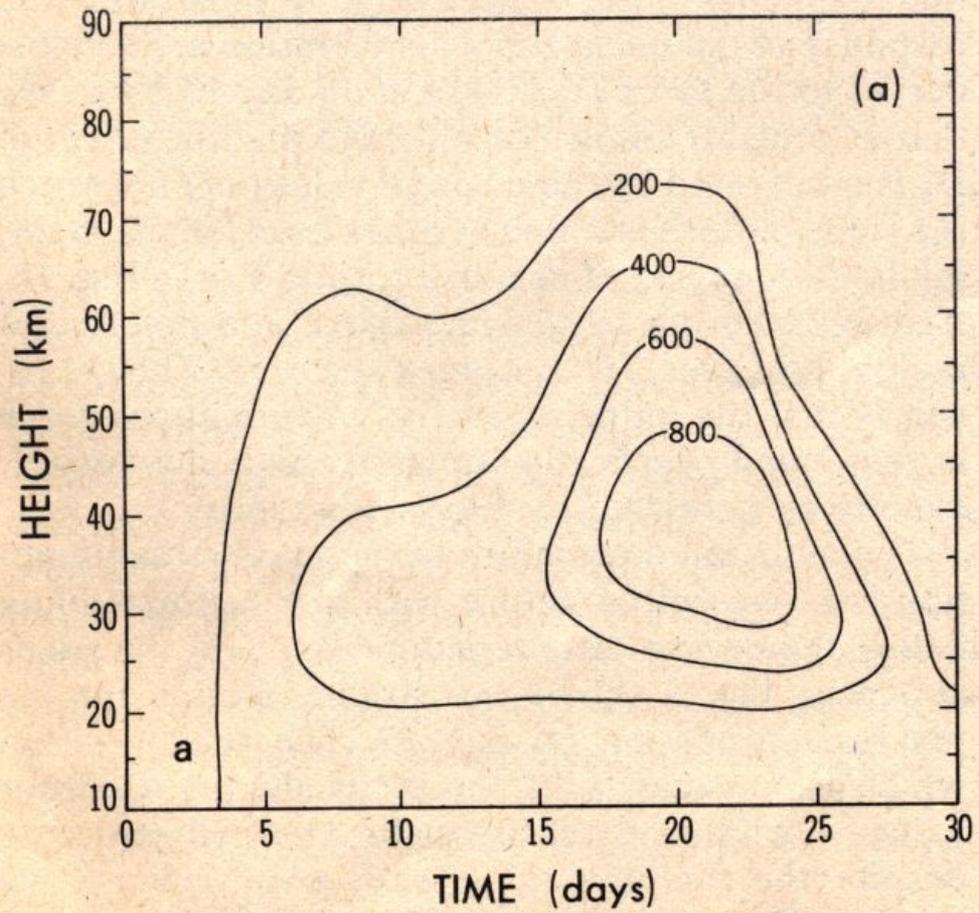
● Equivalent-barotropic dynamics? Storm-track forcing?

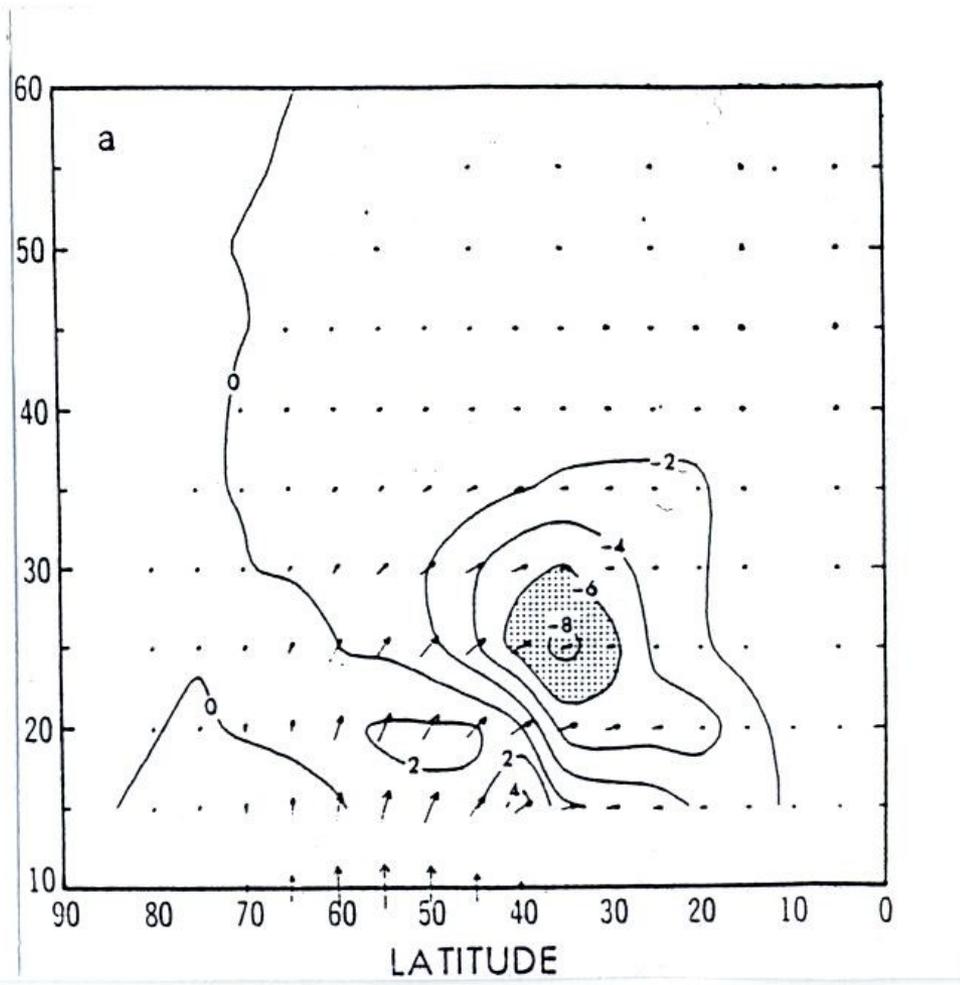
● Linearize about time-mean, zonally-asymmetric state? (Simmons, Branstator, Wallace)

● "Local" resonance? (see my JMSJ review) (1982)

(1982)

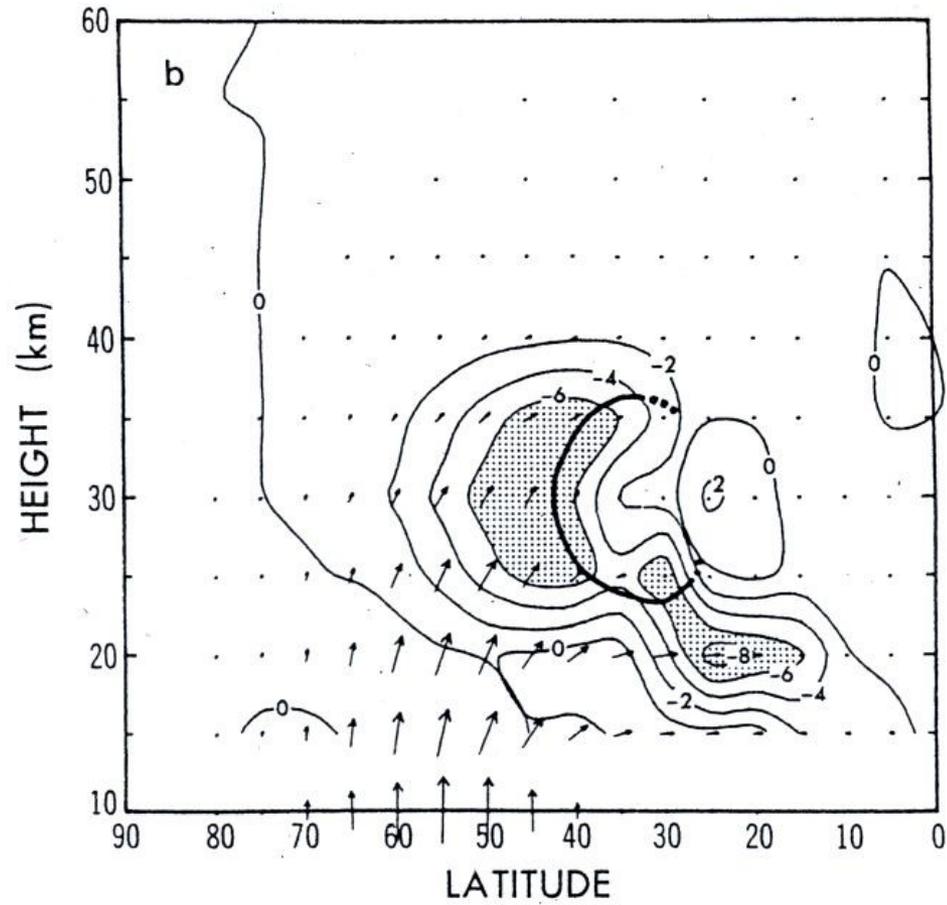




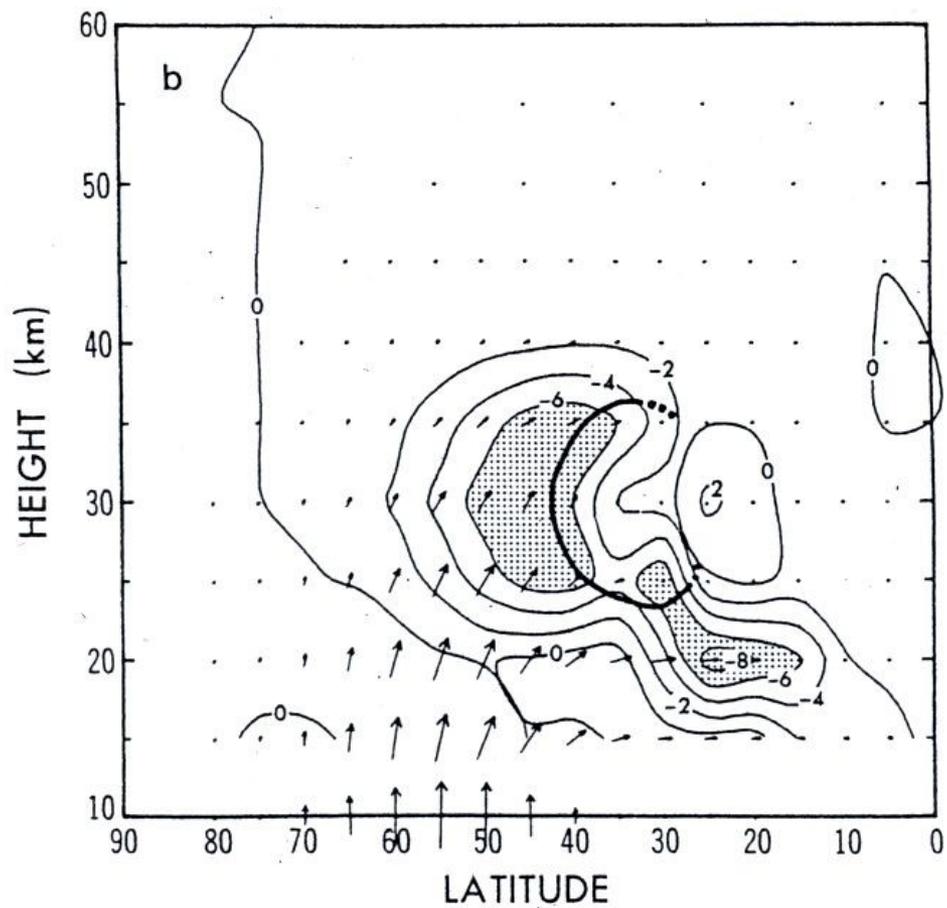


DAY 15

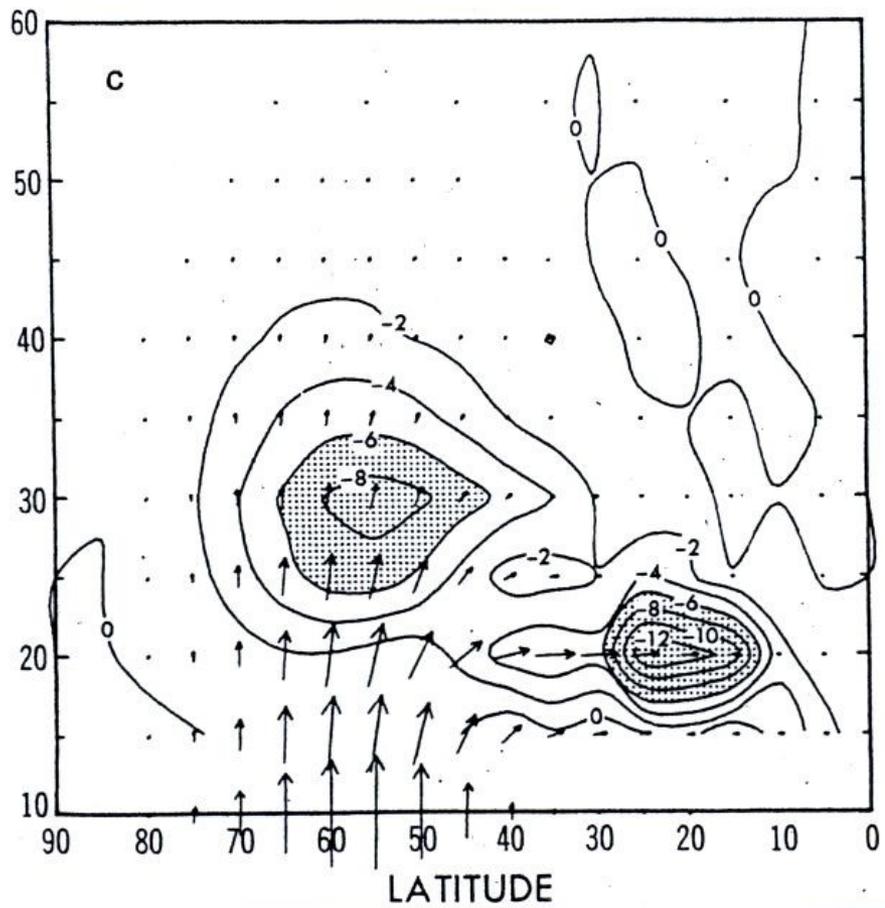
DAY 22



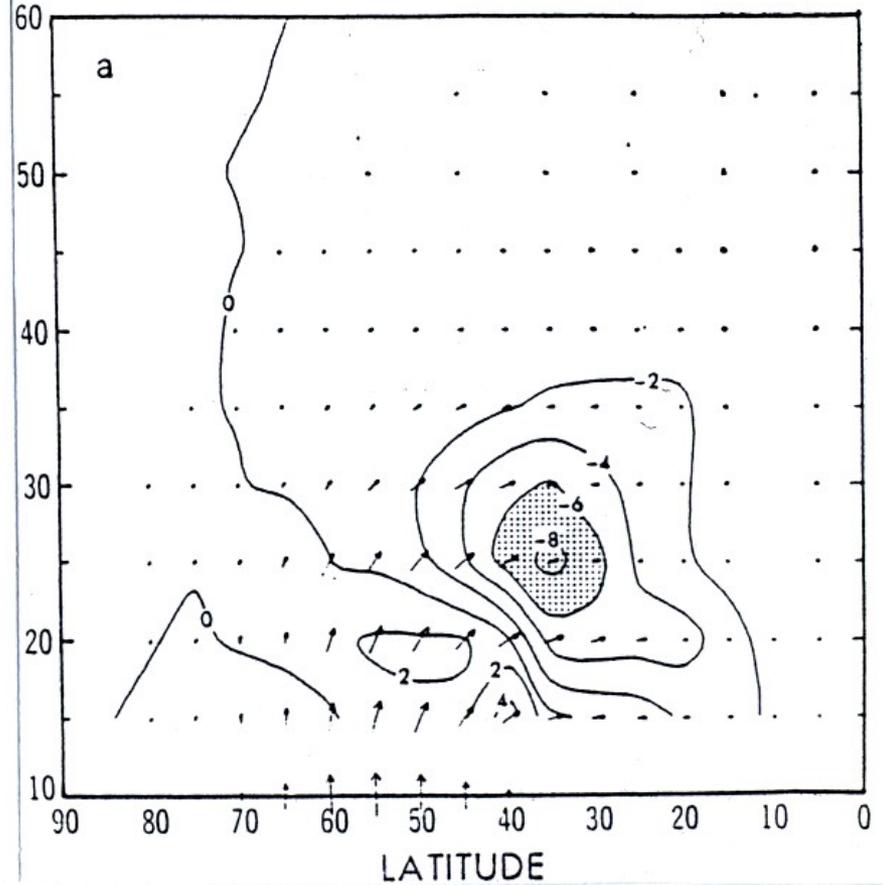
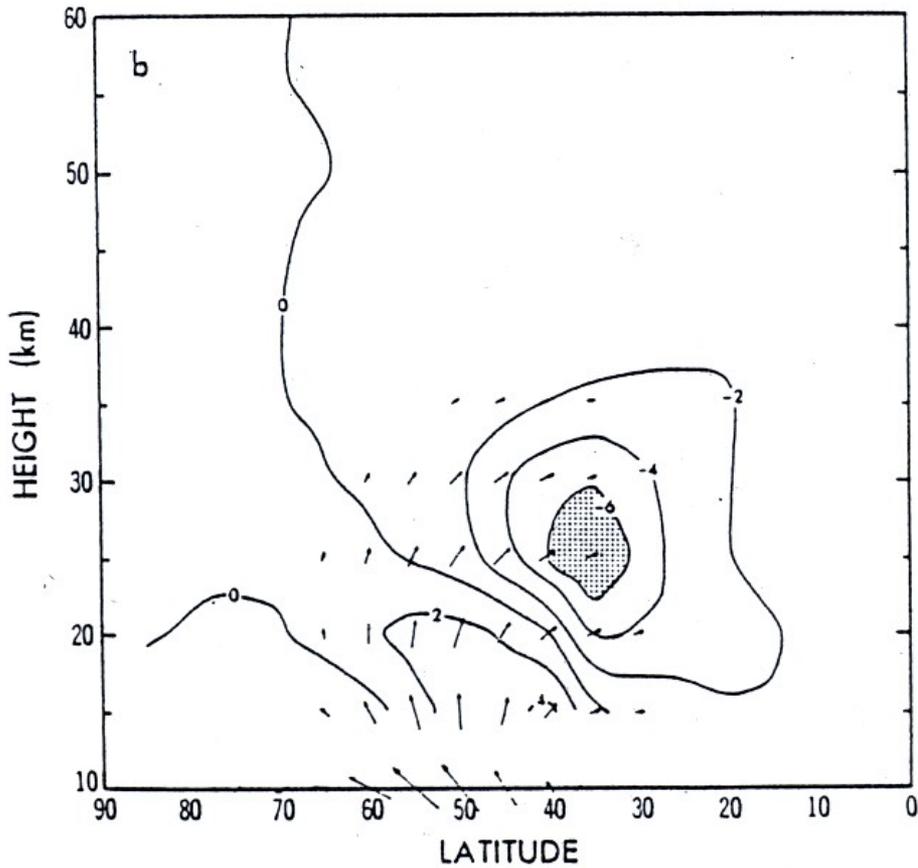
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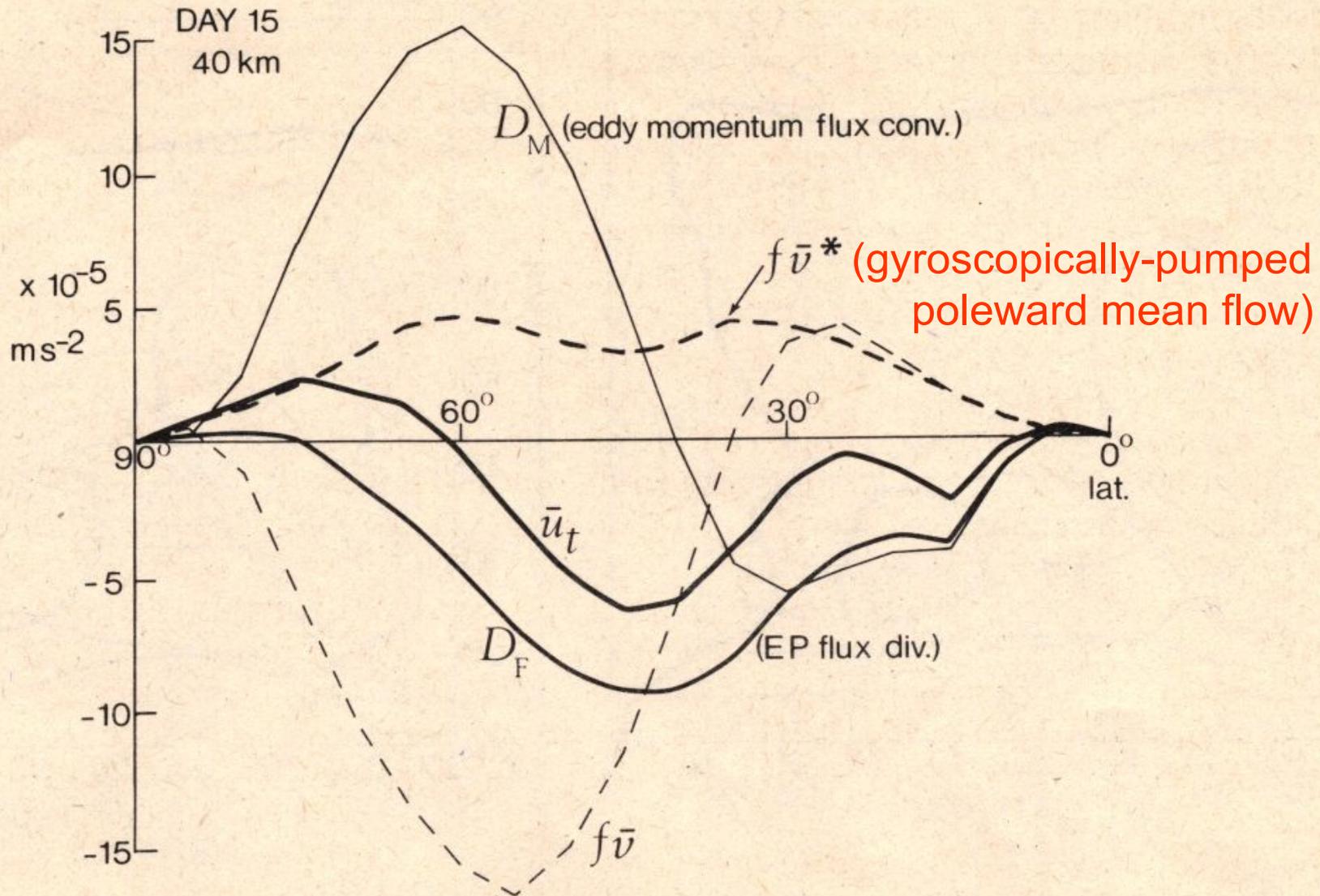
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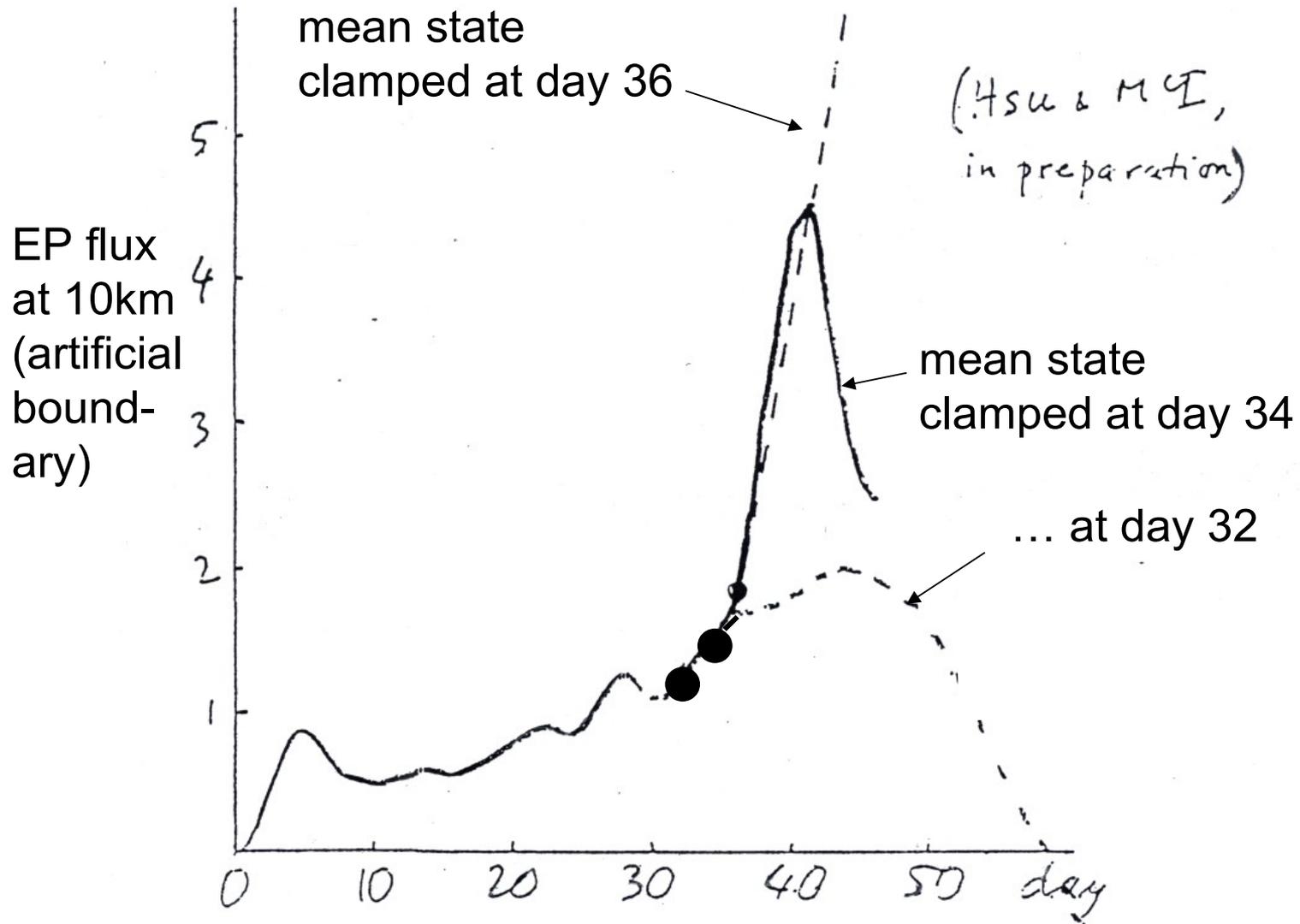
DAY 11



McIntyre 1982, J. Met. Soc. Japan, quoting Hsu (personal comm'n)



**NB:** response **TALLER** than forcing by EP flux div'gce  $D_F$



Some questions from my 1982 JMSJ review:

1. How and why do planetary-wave amplitudes become anomalously large?
2. When they do become large, for what stratospheric conditions is a major warming likely to occur (and why are major warmings relatively uncommon)?
3. To what extent can we use linear planetary-wave theory for the wave structure? And in particular,
4. to what extent can we think of the principal zonal wavenumbers, 1 and 2, as acting independently of each other? In other words, how much can we explain without invoking nonlinear interactions between different zonal wavenumbers?

5. Are wave-reflection and resonance phenomena important or not (*e.g.* to question 1)?

6. Are "critical lines" important or not (*e.g.* to question 2, or to question 5)?

7. Are shear instabilities involved at any stage, and are they relevant to question 1?

8. What quantities should be monitored in order to be able to forecast warmings?

9. To what extent, and in what sense, does the troposphere behave independently of the stratosphere (for the purposes of question 1 for instance), and how should we represent tropospheric-stratospheric coupling in mechanistic models?

Further questions include **whether the Antarctic final warming can be regarded as involving wave-1 resonance...**

Esler-Scott work now makes the clearest case for a resonant response mode **inherently involving the stratosphere and troposphere tightly coupled together...**

A **BIG** remaining problem is how to quantify the coupling, *e.g.* via tropospheric eddy fluxes in storm tracks and other features tied to geography

Early 1980s... a story of scientific good luck: the stratosphere as an outdoor fluid-dynamical laboratory...

Significant events visible because of their large vertical scale, visible even to nadir sounders...

Beautiful early case study:

Clough, S. A., Grahame, N. S., O'Neill, A.,  
1985: Q. J. Roy. Meteorol. Soc., 111, 335-358

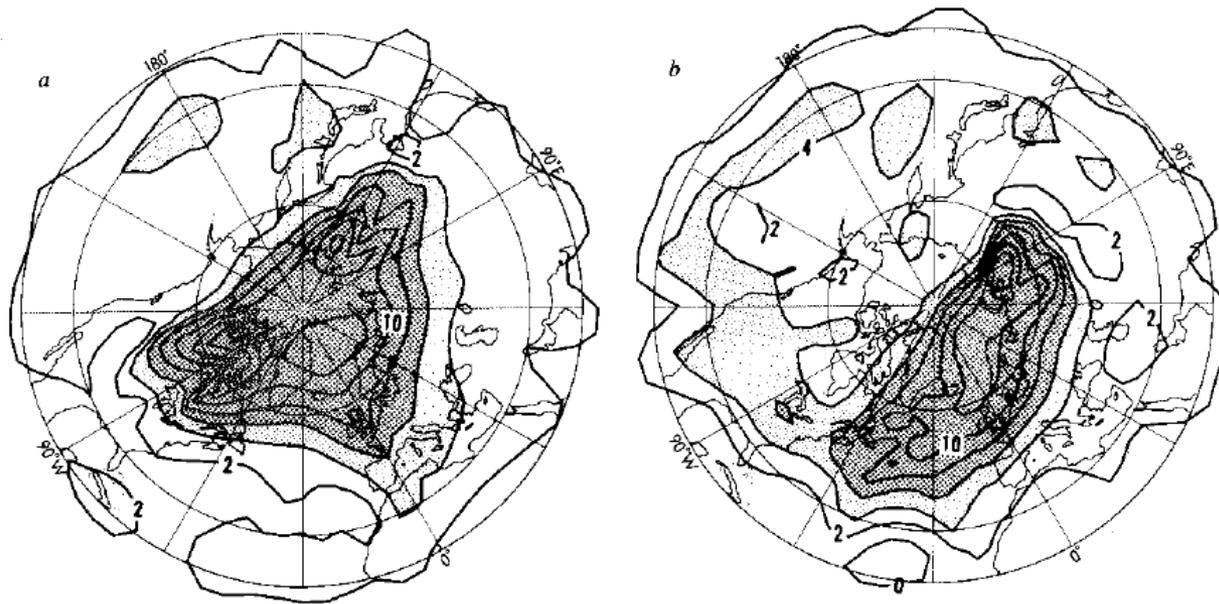
## Breaking planetary waves in the stratosphere

M. E. McIntyre\* & T. N. Palmer†

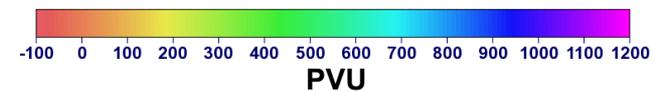
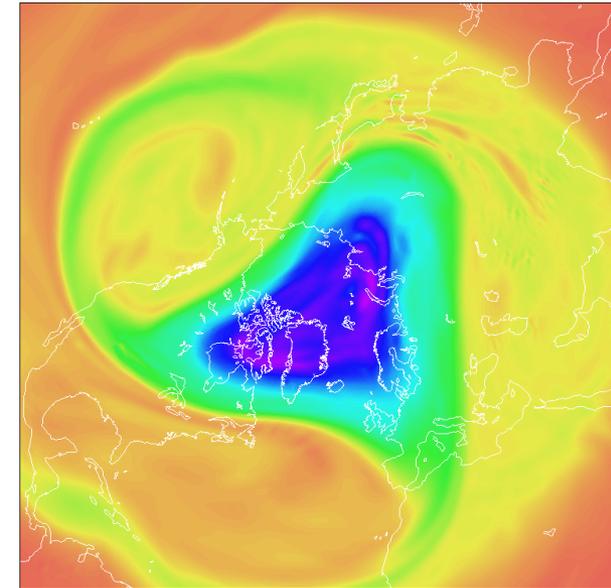
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### Initial state



Potential vorticity at 850K 00UTC 1979/01/17



**Fig. 2** Coarse-grain estimates of Ertel's potential vorticity  $Q$  on the 850 K isentropic surface (near the 10-mbar isobaric surface) on 17 (a) and 27 (b) January 1979, at 00 h GMT. The southernmost latitude circle shown is 20° N; the others are 30° N and 60° N. Map projection is polar stereographic. For units see equation (5) onwards. Contour interval is 2 units. Values greater than 4 units are lightly shaded, and greater than 6 units heavily shaded.

I had been too impressed by the words of an eminent colleague:  
“anyone who tries to compute so highly differentiated a quantity  
as PV from observational data **is a fool.**”

*from old savannah talk:*

(so, e.g., PV mixing has to **fit in** with radiation stress:  
this is **why Rossby-wave breaking can mix PV** so easily)

(BUT Jupiter might be a case of stress governing PV flux...)

**Another question:** any progress in extending idealized perfect-surf-zone models like that of Esler and Scott (2005, J. Atmos. Sci.) to more realistic vertical structures?

Contour-dynamics model with perfectly mixed (therefore perfectly reflecting) surf zone

With a realistic lower boundary condition the system has an “external” or “barotropic” mode. Even within linear theory this has its own built-in “upper reflector”.

Can get strong **self-tuning resonance by vortex erosion**, with modest forcing (quasi-topographic) – big EP uprush!

The authors judge that the wave-2 warming of February 1979 involved simple barotropic resonance, but not the SH wave-2 warming of September 2002., which had more phase tilt leading to a 2-vortex helix.



Indeed, the dynamics is **strongly** nonlinear. There are intimate, promiscuous, non-resonant interactions among many Fourier components. It's also spatially **very inhomogeneous**.

Homogeneous turbulence theory is **inapplicable (!)**

And standard theoretical physics might well say that the problem of understanding the dynamics is hopelessly intractable. **However,**

some of the most important aspects are captured by a very simple, yet powerful idea -- the idea of **PV mixing** (complete or partial).

(It's an old idea, by the way. Its ancestry is traceable back to G. I. Taylor's vorticity-mixing ideas, 1915 onward).

**Why** is this idea powerful? Let's remind ourselves of the main properties of the PV.

It's useful to define the exact PV in a way that applies equally to single-layer and multi-layer, compressible and incompressible, stratified rotating systems:

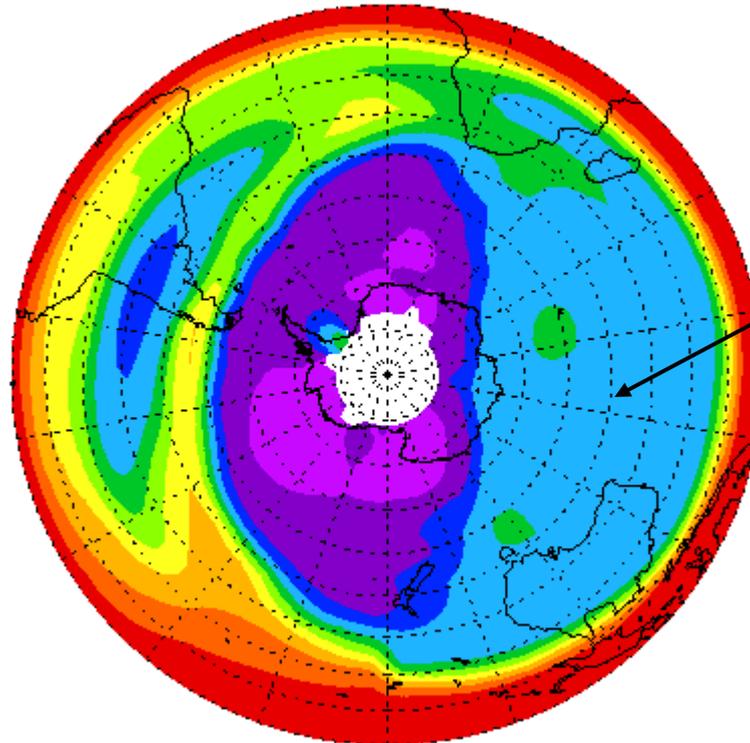
**Momentum transport is intimately associated with PV mixing.**  
(Material invariance says “mixing” can make sense.) **Two ways** to see it:

## **(2) Idealized thought-experiments on PV mixing.**

These idealize phenomena that are observed for real:

Aug. 10, 1997

CRISTA  
N<sub>2</sub>O in upper  
stratosphere,  
courtesy  
Martin  
Riese



This is  
the so-called  
“stratospheric  
surf zone”

For a tutorial on the dynamics,  
websearch “dynamics that is significant for chemistry”