The Acoustics of Stringed Musical Instruments

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Attempts to understand the action of musical instruments at a level having some impact on the problems of musicians and instrument makers must face formidable unsolved problems in psychoacoustics as well as in classical physics. This review outlines what is known about the physical properties of stringed instruments, with emphasis on the violin family, and tries to identify basic outstanding problems where progress may be hoped for within the next decade. One such area is assessment of the widely used simplest idealization of violin behaviour in which the driving at the bridge is assumed to be transverse, and the body is modelled as a 'black box' with one input and one output. Other major problems include the physical nature and psychoacoustical relevance of complex fine detail in the behaviour of the bowed string.

According to tradition, Pythagoras was first guided to the notion that mathematics held a key to understanding nature by observations of the relation between musical intervals and natural numbers. From this seminal position, musical acoustics remained a recognised part of scientific endeavour until the early years of this century, with scientists of the calibre of Rayleigh and Helmholtz making major contributions. However, most of the physicists and applied mathematicians of the next generation turned away from such problems to the new and exciting developments of relativity and quantum theory. Only in recent years has there been a general revival of interest in classical problems, including musical acoustics.

Indeed it has been recently suggested that musical acoustics may again hold a key to progress in one of the frontiers of science, namely the understanding of the human brain. In a recent paper entitled 'Research potentials in auditory characteristics of violin tone', P. C. Boomsliter and W. Creel[1] point out that

> Our understanding of hearing and of the violin can and should move forward together . . . When we study the variations introduced by the 'slip-stick' action of the bow, we are also studying the nature of short-time organization by the nervous system. When we study note tunings in violin performance, we are also studying principles of association in hearing. When we study richness and body in violin tone, we are also studying what makes for continuing attention and satisfaction in human perception. When we study the phrasings of virtuosos, we are also studying the longer organizing systems of the human mind.

This growing awareness of the psychological dimension of the subject in turn affects the type of problem to which physicists address themselves, and also the scope of their enquiries: alongside an examination of the behaviour of instruments they must do the psychoacoustical work necessary to correlate the subjective judgements of the musician with physical effects amenable to measurement and theoretical investigation.[2,3] As we shall see, this work presents challenges that can be even more formidable than those of physics.

We review here recent work on the stringed musical instruments, especially those of the violin family as they pose some of the biggest questions. We emphasize problems that are not dealt with in the excellent recent book by Benade,[3] and we emphasize questions rather than answers: the reader must not expect to find any 'secrets of Stradivari'. Serious research has not yet reached this level of confident prediction. It is unfortunate that the popular image of violin acoustics has been largely moulded by purveyors of pet theories on this subject (see for example chapter 5 of Heron-Allen[4] entitled 'The violin, its vagaries and variegators').

THE STRING AND THE SOUND BOX

The physical behaviour of a stringed instrument can be examined under three headings: first the behaviour of the stretched string whose vibration is controlled in one way or another by the player, second the response of the wooden sound box of the instrument and the neighbouring air in response to the string motion, and third the radiation of sound, almost entirely from the sound box and generally involving a complicated directional dependence. These three cannot be treated entirely separately,

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because there is a back-reaction of the body vibrations on the string which in the bowed instruments has an important effect on the 'feel' of the instrument to the player. The body vibrations are in turn affected by a radiation reaction. We deal here mainly with the first and second topics; aspects of the third topic concerning directional radiation patterns are covered in an earlier review by Cremer[5] and elsewhere.[6]

Though the behaviour of a stretched string which is plucked or struck is well understood, the behaviour of a bowed string is much more complicated. The motion was understood in outline by Helmholtz as far back as 1860, and much additional detail was elucidated by C. V. Raman early in this century. However, many of the musically all-important finer details have only recently begun to be explored. Of particular interest are the tolerance ranges in the various parameters under the player's control for a musically acceptable 'steady' note to be produced, and also the length and nature of the string transients involved in vibrato, 'attack' and 'articulation', to which the ear is particularly sensitive. The 'steady' case has been investigated to some extent already, the transient case hardly at all. Both present outstanding problems for the future.

The behaviour of the sound box is a concern of the instrument maker, and much information is available from master luthiers who have been prepared to talk about their ideas and methods. Especially important contributions have come from scientifically minded makers who take extensive acoustical measurements on components of instrument bodies at various stages of construction. These results, and theoretical investigations based on them, are apparently beginning to yield some of the essential characteristics of good instruments, although not the distinguishing features of the very best ones. (Indeed it is questionable whether 'best' has any unique meaning.)

One important fact to emerge is that at present we do not know enough about the properties of wood: we need to know the elastic properties and especially the internal damping behaviour in more detail than was previously realised. Useful light is being shed on this problem by a recent effort to develop a carbon fibre composite material to duplicate the necessary properties of Norway Spruce, *Picea excelsa*, the wood used for the soundboards of almost all stringed instruments.

Understanding musical instruments involves making theoretical models of their behaviour. Before considering in detail various models that have been proposed for the bowed string instruments, we should ask a basic general question: what constitutes an adequate model in musical acoustics?

MODELS AND RESEARCH STRATEGIES IN MUSICAL ACOUSTICS

From the music of the spheres[7] to the Glass Bead Game,[8] it has been common to speak of connections between music and mathematics. However, musicians and scientists have by no means always cooperated fruitfully in the study of musical acoustics. Musicians tend to think that science has nothing to tell them, while some physicists tend to assume that the study of instruments and concert halls involves no more than straightforward measurement. In fact, nothing could be further from the truth: recent developments in psychoacoustics,[2,3] as well as celebrated disasters in concert hall design, have emphasized the subtlety of the human ear–brain system as well as modifying some of the older established ideas about the mechanisms of hearing.

Vision involves similar subtleties, and Fig. 1 provides an excellent illustration. The reader will have no difficulty in seeing a Dalmatian dog, with a wealth of information about what it looks like and what it is doing. Now one could readily measure such things as statistical distributions of spot sizes and spacings in the picture, or the chemical composition of the printing ink, and it is clear that such data, however elaborate, would not reveal even the existence of the dog. Indeed, it does not take much thought to discover the practical impossibility of writing a general purpose computer program which would recognize the presence of the dog out of the myriad other possibilities.[9]

Similarly, while acknowledging that standard acoustical measurements have their uses, we must

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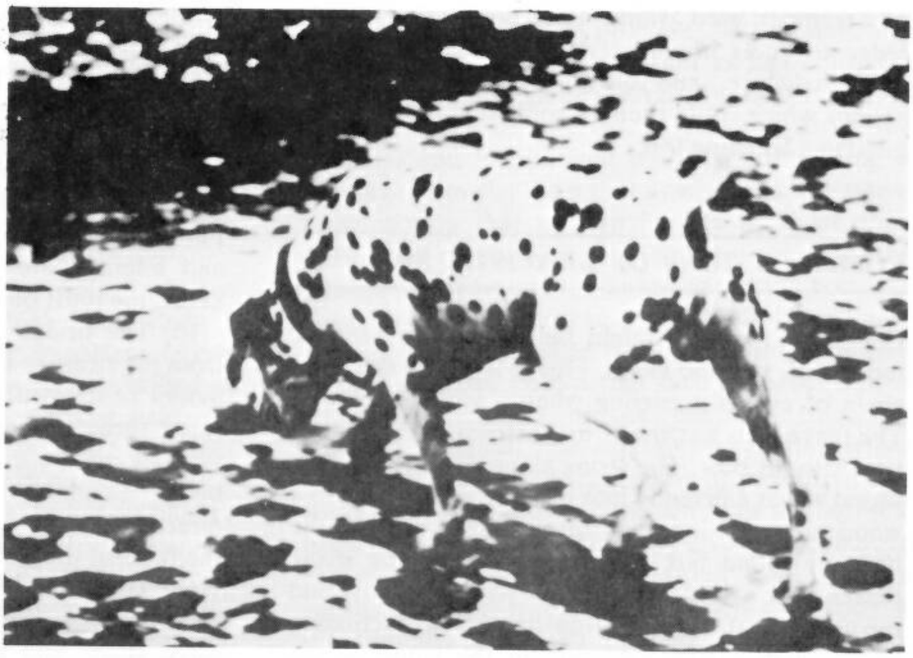

Figure 1. Photograph by Ronald C. James. (Reproduced from *The Intelligent Eye* by R. L. Gregory, by permission of Weidenfeld and Nicolson Ltd.)

guard against the assumption that such measurements can capture everything significant about a musical sound. Again, it is clear that very small changes to Fig. 1 could remove or drastically change the dog, and this is equally true of musical sounds: we must not assume that small physical changes necessarily produce small perceptual effects.[1,3] So when we ask the question 'what is an adequate model in musical acoustics?', one thing is plain: the final test of 'adequacy' must come from the ears of the listener, and the fingers of the player. Lacking as we do a sufficient understanding of the auditory system, we should aim to listen to, and play, our models.[10]

The ability to recognize the dog in Fig. 1 is thought to depend on a hierarchy of 'feature detectors' in our neural signal processing system. Among those simple examples for which direct neurophysiological evidence is available are visual nerve cells which respond specifically to straight lines or edges at various inclinations.[11] Past experience is important: feature detection apparently involves matching against internal paradigms.[1,2] In the auditory system, feature detectors can be roughly classified according to the time-scale over which they operate.

The shortest time scale is that evidenced by pitch discrimination: vibration period differences of tens of microseconds are involved.[2,3,12] The opposite extreme is encountered in the perception of melody and rhythm, involving time scales from fractions of a second upward. In between are what we can term phenomena of 'intermediate time scale', such as starting transients and vibrato, where significant changes occupy tens of milliseconds. These last are the time scales of consonants in speech, and are also of prime importance in the recognition and quality assessment of musical instruments by their sounds. Correct description of intermediate time scale phenomena is thus an extremely important requirement for models in musical acoustics.

The foregoing remarks necessitate a critical attitude to two research approaches which are frequently suggested. On the one hand, one could take an instrument and try to measure all the relevant data about its physical behaviour: such blind measurement is usually sterile since knowledge about what is 'relevant' to the ear is so limited. On the other hand one could try to simulate on a computer the entire chain of acoustical events from the actions of the player to the signals at the eardrums of the listener: such large scale numerical modelling is, however, much more limited in scope than is sometimes supposed. A sufficiently accurate simulation of a musical instrument in its acoustical environment is well beyond the reach of present day computers; and even if it were not, it would be impossibly cumbersome to use as a research tool because of the volume of information that would be produced.

To make progress we must divide the chain of events into components and study these individually and in simple interactions before trying to comprehend everything at once. Such study should combine restricted model building with measurement and listening tests. Two examples may be cited as illustrations, each successful in its own way. The first is the work of Benade *et al.*,[3] in which a simple theory was used to interpret measurements of the properties of a wind instrument. The theory then suggested that certain small changes in the tube bore and hole geometry would improve the playing properties of the instrument, and subsequent playing and listening tests confirmed this. The second is the work of Risset and Mathews,[10] who analysed sounds of a trumpet, used available psychoacoustical knowledge to guess important features, including transient structure, and on this basis resynthesized sounds which could then be compared with the original in a listening test.

COMPLICATIONS OF REALISTIC MODELS

Consider now what might be involved in realistic models of, say, the violin. Figure 2 recalls the basic chain of events occurring when a violin is played. The player sets a string of the instrument into vibration with his bow. The string alone does not radiate sound waves efficiently into the surrounding air, so a wooden resonating structure is used, namely the body or sound box of the instrument. The string passes over a bridge in contact with the body, and the movement of the string imparts a fluctuating force which excites vibration in the wood of the body. This in turn generates sound waves in the surrounding air which radiate away to reach the ears of the listener. Even if we leave aside the *terra incognita* of the biological links in the chain, a closer look at the purely acoustical aspects reveals a long list of complications:

(a) The bow which excites the string is itself a dynamical system, whose properties influence the string's behaviour.[13] This fact, well known to players, has received relatively little attention from physicists.

(b) While the visible motion of the string is predominantly transverse, there will inevitably be torsional[14] and longitudinal[15] motions as well, greatly complicating the driving force at the bridge. Even for the transverse motion, the linear wave equation familiar from acoustics textbooks is by no means the whole story for at least two reasons: first, a real string is anharmonic because of slightly yielding terminations[3] and finite flexural stiffness,[16] and second, the string can be played at sufficient amplitude to introduce nonlinear effects.[15,17]

(c) Many of the problems of real musical interest are concerned not with the relatively simple motion of the string during the middle, nominally 'steady', portion of the note but with the complicated transient motions at the start and finish of the note. Furthermore, even 'steady' notes may contain short and intermediate time scale variations of several kinds, the most obvious of which is vibrato.

(d) The bridge does not merely transmit forces from the string to the body passively: it has vibration modes of its own, many of them involving complicated three-dimensional motions,[18] and these can strongly influence the eventual driving force on the body.[19] In addition, there will be a certain amount of direct radiation of very high frequency sound from the faces of the bridge.

(e) The geometry of a violin body does not lend itself to simple theory: the combination of a complicated shape and a constructional material, wood, having a large number of independent elastic and viscoelastic constants makes detailed modelling cumbersome. Also, despite the small amplitudes of typical motions in the body, it is possible that some nonlinear effects may have audible consequences; for example, effects associated with the purfling inlaid around the edge of the front and back plates of the instrument.[20]

(f) The strings must be regarded as part of the body for the purpose of analysing its mechanical behaviour. As Schelleng[14] has pointed out, the longitudinal compliance of the strings, especially in the case of metal strings, will substantially change the way the body vibrates.

(g) The vibrations of the body and bridge can significantly affect the motion of the string. An extreme example is what musicians call a 'wolf note', an unpleasant stuttering effect resulting from the attempt to play a note falling very near a strong resonance of the body. In a less spectacular way,

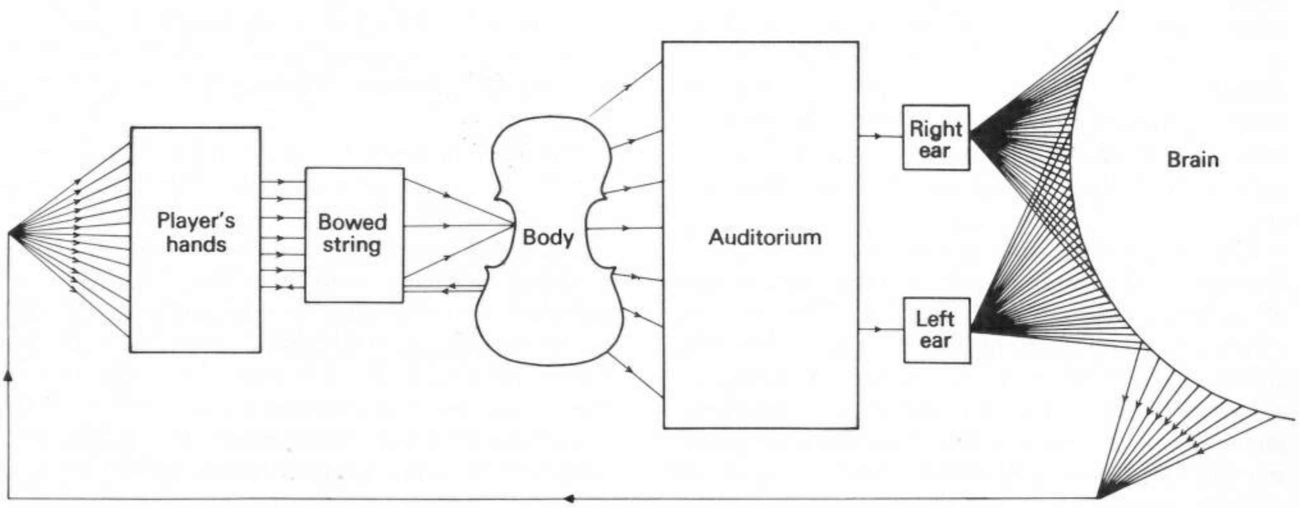

Figure 2. Schematic indication of the chain of events when a violin is played. The picture fails to suggest some of the important ingredients, such as slight movements of player or listener in the auditorium; see text.

back reaction of the body and bridge on the string influences the ease with which any note may be established, and the length and nature of its starting transient.[21]

(h) The air motion in and around the instrument couples to the motion of the body.[22,23] This coupling is generally much stronger than that between the body and the string: the body is, after all, designed to radiate sound.

(i) The radiation properties of a violin body are very complicated, especially at high frequencies.[5,6] There are many possibilities for important effects here, since the ear–brain system is very skilled at using directional information in a sound field; for example, this contributes to the ability of a listener to follow a conversation in a crowded cocktail party. The fluctuation of the directional radiation pattern due to vibrato might, for instance, be important; if so, this would help explain why conventional recording techniques seem unable to capture the sound quality of a violin played in a concert hall. Benade[3] has demonstrated that the ear is sensitive to the directional radiation patterns of wind instruments played indoors; changes in these patterns are perceived by the listener as changes in subjective tone quality.

(j) The room in which an instrument is played has many vibration modes in the audible frequency range: even a domestic living room has tens of thousands. Consequently, acoustical measurements with a single microphone show wild fluctuations when the microphone or any other object is moved slightly. This complexity, far from being a problem for the ear, is put to positive use;[3] for example, blind people learn to use the information to 'hear' the positions of obstacles in a room.

This list is by no means exhaustive. It suggests that the first level in a hierarchy of models must inevitably involve drastic simplification.

THE RESPONSE-CURVE MODEL

In the first instance we take the string motion as given, and concentrate on the action of the violin body, for this purpose regarding the bridge as part of the body. Much of the early experimental work on the subject took this approach, since the main interest then was in seeking scientific ways of distinguishing between different violins, especially between good ones and bad ones, on the basis of the sounds they made. On the grounds that the motion of the string is much the same on any violin such differences were naturally sought in measured characteristics of the violin body. When we seek the differences betwen playing properties of instruments however, it becomes essential to model the string motion, and we consider such problems in detail in later sections.

The very simplest idealization is what may for brevity be called the 'response curve model'. To reach this, we first put the complexities of the radiation and room acoustics problems on one side, and consider the sound of the violin as heard from a single, fixed microphone position. Next, we suppose that the force on the bridge resulting from the torsional and longitudinal motion of the string is sufficiently smaller than that arising from the transverse motion that we can neglect it. If we then make the relatively minor assumptions that the radiation process and the behaviour of the body are linear to sufficient accuracy, we are left with a linear 'black box' with a single input and a single output.

Now it is well known that such a system can be completely defined by its complex frequency response function, that is a single function which specifies the amplitude and phase of the output sine wave when the system is driven with a sinusoidal force of unit amplitude and any given frequency. In experimental work on the violin family, as in specifications for hi-fi equipment, it is customary to plot only the amplitude, although this may well not be adequate.[24]

An example of a response curve measured from a Stradivarius violin (the 'Titian' of 1715) is shown in Fig. 3. In this case, the instrument was placed in nearly anechoic surroundings to minimise effects from the behaviour of the room. A transverse force, whose amplitude was held constant as frequency varied, was applied electromechanically to the violin bridge. Peaks in the response curve indicate either frequencies at which the radiation pattern of the instrument happens to favour the microphone direction, or resonances of the violin body, including the air in and around it. Some of the resonant vibration patterns of instrument bodies at such peaks have been explored experimentally by various techniques.[25–28]

The lowest peak in Fig. 3, around 270 Hz, is a typical feature of response curves of reasonably good instruments.[25,29] It has the distinctive property of disappearing when the f-holes are blocked[30] and shifting to a lower frequency when the instrument body is filled with carbon dioxide.[31] The air cavity within the body, together with the f-holes, is acting somewhat like a Helmholtz resonator, modified by coupling to body motions.[22,32] (In guitars, the modification is drastic.)[33] The higher air-cavity modes do not give rise to such easily identifiable features in response curves, but their possible role has been studied by Jansson.[34]

Many response curves have been published,[31,35,36] but unfortunately some of them bear no close relation to the action of the instrument when excited by its strings because the driving force was not held constant as frequency varied.[37] Comparisons of response curves obtained by different excitation methods have been given by Bradley and Stewart[38] and Hutchins.[39] Refinements aimed at minimizing directional radiation effects so as to concentrate on the resonances of the violin include the use of many microphones,[36] and the use of a reverberation

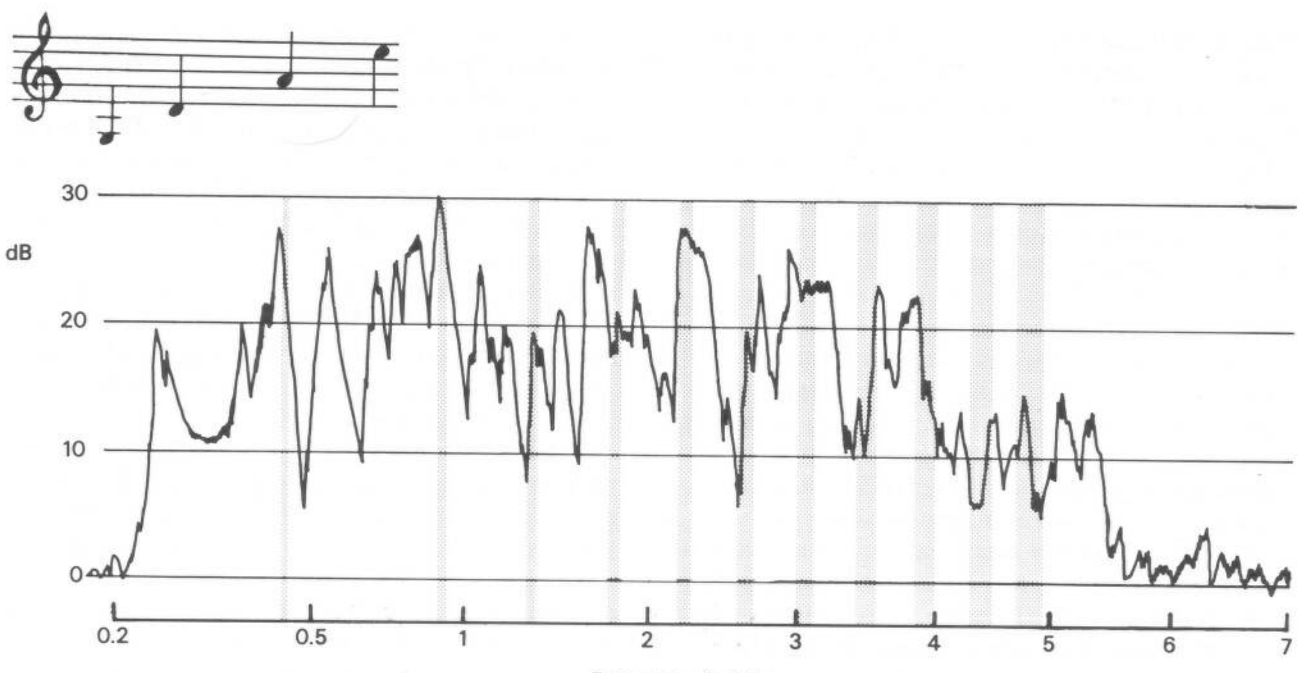

Figure 3. Response curve of a Stradivarius violin (the 'Titian' of 1715, after Saunders[31]). Sound intensity at one microphone position is plotted against frequency. The musical notes shown are those to which the open strings are tuned. The shaded bars indicate the range of variation of harmonics of the note 440 A during a vibrato cycle.

chamber in place of an anechoic one.[30,40]†A related measurement on our black box which focuses on the aspects of body behaviour relevant to the dynamics of the string motion is the 'input admittance curve', a plot of the amplitude of the transverse bridge motion which results from the given driving force.[5,30]

The 'spikiness' of violin response curves has a simple but crucial consequence. When a note is played with vibrato on the instrument, the frequency of each harmonic moves up and down over a range of up to a semitone, as suggested by the shaded bars in Fig. 3. It is plain that the response curve can have considerable variations within such ranges, different for each harmonic. Thus in addition to frequency fluctuations, the output waveform from the black box will exhibit appreciable fluctuations in the spectrum of harmonics, with some increasing in strength while others decrease. This fact was first pointed out by Fletcher and Sanders,[41] who also showed by simple listening tests that the effect is essential to the characteristic subjective quality of violin sound. The listener does not generally notice the spectral variations as such; rather, these are 'feature detected' in terms of tone quality – an intermediate time scale effect. If vibrato is played in 'slow motion' it is no longer feature detected in this way, and the spectral and frequency fluctuations become clearly audible as such.

Response curves and input admittance curves can be readily calculated for simplified theoretical models of instrument bodies, and these calculations compared with measurements such as Fig. 3. Such comparison enables the model to be adjusted to bring it into closer agreement with the observations, in the process giving insight into the physical meaning of specific features of the measured curves. Beldie,[30] for instance, used a simplified mechanical model of the violin body to elucidate its low frequency behaviour. Another such model is currently being studied by Cox and Fellgett.[42] Reinicke[19] used a similarly simple model of the bridge dynamics, and convincingly showed the importance of bridge modes to the response above 2 kHz.

Schelleng,[22] in an earlier paper which has become a classic in the field, used an electric circuit analogue model to elucidate a wide range of questions. These included the general shape of the response curve, the role of the lowest 'air' mode in promoting efficient radiation above its resonant frequency, and also the mechanism of the 'wolf note' to which we shall return briefly in a later section. In addition Schelleng established rules for scaling of different instruments of the violin family to give approximately the same relative response curve with instruments of different tunings.[43] This throws light on the differences between the conventional violin, viola and 'cello, which are not scaled in this way.

Attempts have been made to associate many of the subjective characteristics of instruments with recognisable features of response curves – we have already mentioned the 'spectrum vibrato' effect. Another point which seems fairly well established[29,35,36] is that relatively strong response at the lowest frequencies is a characteristic of those violins traditionally most admired by musicians. 'Relative strength' is a matter of some delicacy: 'the master maker', it appears, 'must conserve every

†A new and promising experimental approach has recently been initiated by Arnold and Weinreich.[79]

decibel with miserly care'.[44] It is of interest that the so-called 'soundpost' inside the body, known in French as the 'soul' of the instrument, plays a crucial role in the low frequency response. Its asymmetric position promotes the oscillatory volume changes which the air in the cavity must undergo, as every acoustician knows, if there is to be efficient radiation at low frequencies.[5,45–47]

Attempts to discern further subjectively important features of response curves run into two severe difficulties. The first difficulty is the obvious one, that with real instruments it is never possible to vary one parameter while keeping everything else constant; any two real instruments differ in a large number of ways. The second difficulty is that of communicating subjective impressions in words: thus Meinel,[35] in the course of a summary of many years' work on response curves, states that 'small amplitudes near 1500 cycles per second prevent a very nasal character', and it is clear from the context that this nasal quality is considered undesirable. The word 'nasal', however, has different meanings for different authors: to Leipp[48] it is one attribute of the desirable old Italian sound.

What is needed is the type of experiment indicated in the third section, and this has been very elegantly provided by Mathews and Kohut.[49] They converted the transverse motion of a set of violin strings into an electrical signal, and then passed this signal through a set of electronic filters giving a spiky response function like that of Fig. 3 before listening to it through a loudspeaker. This approach opens two very profitable lines of enquiry. First, the electronic response function can be varied at will, keeping exactly the same recorded player input, so that statements like Meinel's can be put to controlled listening tests. Second, the strengths and limitations of the response curve model itself are revealed by the extent to which the hybrid 'violin' can be made to sound in the hands of a competent player like a monaural recording of a good real violin.

The conclusion of Mathews' and Kohut's pilot study was that a reasonable first approximation to the sound of a real, monaurally† recorded violin could indeed be produced. The subjective quality was fairly sensitive to the sharpness of the resonance peaks: this appears attributable to the effects on vibrato and on other transients.[50] But none of the response functions used by Mathews and Kohut produced a result good enough to fool trained musicians' ears. It is not yet clear whether this failure is due primarily to unrealistic features of the response function, or whether it points to major inadequacies in the response curve model itself, such as the neglect of string motion other than transverse.

One extension of Mathews' and Kohut's experiment is being tried with some success by Gorrill,[51] who has incorporated the loudspeaker playing the processed string signal into the back of the instrument being played, a viola in this case. Even though this necessitates a small loudspeaker, the resulting sound seems rather more realistic than any that Mathews and Kohut have yet achieved. One possible reason is the more realistically variable radiation pattern of the hybrid instrument, both as the frequency changes and as the player moves during performance. (See (i) and (j) on p. 161.)

The more realistic feedback to the player may also be significant. Another possibility is that, although Gorrill's viola body is strongly strutted internally to minimise vibration, there is a small amount of 'live' sound from the instrument which may be audible, in particular in the starting transients of notes to which the ear is so sensitive. The systematic blindfold listening tests required to decide this last point have not yet been made, as far as we know.

Much more work could, and hopefully will, be done on this key experiment. A very basic question is: what kind of changes in response curves are *least* noticed by listeners? This question could in principle be studied by the powerful 'objective listening test' techniques used in modern psychoacoustics,[52] although the magnitude of the task is daunting. A start has been made by Jansson and Gabrielsson,[53] using the technique of 'long-time-average spectra.'[40] A related question is suggested by the violinist's need, especially in chamber music, to command the greatest possible range of perceived tone quality. At least two factors contribute to producing such a range. First, some violins may produce a greater variety of waveforms than others, for instance through the mechanisms to be discussed on pp. 168 and 169. Second, for a given range of bowed string input waveforms, some response curves more than others may bring the range of output waveforms into a part of 'perceptual space'[52] where the ear notices the changes most.

INTERPRETING THE VIOLIN MAKING TRADITION

Instead of trying to find out how we want a violin body to behave, we now look at the instrument-making tradition to see how a skilled luthier tries to make it behave. This is a very large field, and there is by no means perfect agreement between makers on how the main problems should be handled, or indeed on what the main problems are. We shall concentrate on a single example which has shown the sort of work which can be done in this area: we examine the methods used by makers to arrive at the best distribution of thickness in the back and front plates of an instrument. This process is apparently regarded by most makers as a major one in determining the sound and the playing properties of the finished instrument, and has been investigated in some detail by makers who use acoustical measurement techniques, notably Meinel[25,35] and Hutchins.[54]

† and anechoically

We describe a commonly used method: once the outside shape of a plate is completed, the violin maker hollows it out to achieve a thickness distribution approximating a standard pattern. However no two pieces of wood are identical, so no one pattern can be perfect for all plates. Thus the luthier makes small adjustments, occasionally holding the plate up lightly between finger and thumb and tapping it with a finger. He listens carefully to the note produced, and more particularly to the quality of the note, and he tries by his thickness adjustments to achieve what has been described as a 'clear, full ring'. He might do this for several positions of holding and tapping, obtaining in this way more than one note.

In more scientific terms he makes heuristic observations of the frequency, damping and mode shape of one or more vibration modes of the plate, and adjusts the thickness distribution apparently to minimise the damping of these modes. Some makers are also concerned to place the frequencies of certain modes in a particular place or in a particular relation to one another. A remarkable feature is the sensitivity of the method: sometimes a difference can be heard as a result of removing just 0.1 mm. of wood from a few square centimetres of a plate of some 3 mm. thickness.

Hutchins[54] comes to the surprising conclusion that subjectively good results in the finished instrument can be achieved by confining attention to a few of the gravest modes of the plates in their unattached state. Their frequencies cover only a small part of the audible range, and in any case the plates behave quite differently under the different boundary conditions in the assembled instrument. Thus it would seem that the free plate modes are in some way acting as barometers for the behaviour of the plates under fairly general circumstances, and if this could be understood we might learn much about desirable behaviour of violin bodies. Some preliminary guesses about such barometer effects have been made on the basis of modelling simpler systems than violin plates.[55]

The extensive measurements of Hutchins have revealed much detail about the changes in the vibration properties of plates over the entire audible frequency range as such plate tuning operations are carried out. Three main methods have been used to this end. One is to measure frequency response curves of the plate by driving it sinusoidally with an electromagnetic transducer and detecting the resulting radiated sound at a standard microphone position in a standard room.[56] The other two are ways of visualizing individual vibration modes of the plate, to see how their frequencies, shapes and amplitudes vary.

The first method is the classical one of observing Chladni patterns: the plate is caused to vibrate in the required mode, and powder is sprinkled on it to make the nodal lines visible. The second is the much more modern technique of hologram interferometry, used for studies of violin plates and complete instruments by Stetson and Ågren,[26] Jansson, Molin and Sundin,[27] and Reinicke and Cremer.[28] Figure 4 shows the Chladni pattern and the hologram of the same mode of a violin top plate.

The results obtained by these methods are sufficiently complex that some theoretical insight is needed to interpret them usefully. When one tries to do the theory of plate vibrations it becomes apparent that, quite apart from the complications associated with the geometry of the violin, great problems are posed by the anisotropic structure of wood. Whereas isotropic materials have only two independent elastic constants, wood has nine in general, and at least four are significant for flexural vibrations of thin plates. While all nine constants have been measured for a few specimens (not generally wood of interest to instrument makers: see, e.g., Hearmon[57]), most large programmes of measurement list only two.[58]

In addition, since makers feel that damping is of crucial importance, the viscoelastic damping constants corresponding to each of these elastic constants should be measured before a relevant theory is constructed. Information on these damping constants is even more limited. We should note that varnish and other treatments of the wood will modify the viscoelastic constants,[44] and some makers partially varnish their plates before final tuning.

One interesting experimental approach to this problem is the recent effort by Haines[59] to tailor-

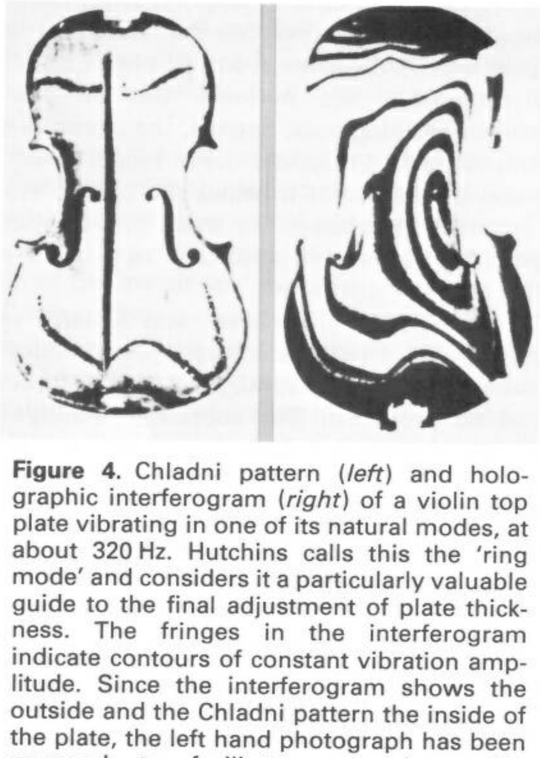

Figure 4. Chladni pattern (*left*) and holographic interferogram (*right*) of a violin top plate vibrating in one of its natural modes, at about 320 Hz. Hutchins calls this the 'ring mode' and considers it a particularly valuable guide to the final adjustment of plate thickness. The fringes in the interferogram indicate contours of constant vibration amplitude. Since the interferogram shows the outside and the Chladni pattern the inside of the plate, the left hand photograph has been reversed to facilitate comparison. (By permission of C. M. Hutchins and K. A. Stetson). For a useful comparison of interferograms for well- and badly-adjusted plates, and related acoustical measurements, see Ref. 56.

make a synthetic material which reproduces the relevant properties of spruce, the wood used for the soundboards of all stringed instruments. The properties of man-made materials can of course be manipulated over a wider range and in a more controlled way than can be achieved with wood. Nevertheless it is remarkably difficult to imitate even a few of the properties of spruce: the reason for the traditional role of that wood in instrument making is its almost unique combination of extreme properties.[22]

The only material currently available which combines the low density, high along-grain stiffness and low damping of spruce is a carbon fibre composite used in a sandwich construction with a light core material such as cardboard or balsa wood. A violin and a guitar have been constructed by Haines and his collaborators with soundboards of such a material, and both have been surprisingly successful.[59] We can hope for much more progress in the near future on this approach, not least because it has commercial possibilities, until recently a rare occurrence in musical acoustics.

As an example of the sort of theory which can be done when the viscoelastic constants are known, the authors[55] have explored the simpler problem of the effect of thickness perturbations on the damping of vibrations in isotropic materials. This has given some clues about the relevance to the finished instrument of the behaviour of the plates before assembly, and about some of Hutchins' other observations.

The results can be represented in pictorial form, and we illustrate this in Fig. 5. The first column shows the four lowest modes of a constant thickness square plate of isotropic material with free edges. The pictures show the plate in perspective with its vibrations frozen as by a stroboscope. The amplitude is exaggerated for clarity. Columns two and three indicate what happens to the frequency and damping of these modes when a small perturbation of thickness is introduced. If a small amount is removed from the plate in a region where the column two function is positive, the frequency of that mode will drop, while if the column three function is positive there, the damping will rise. This applies to each mode separately, so that a large enough set of these pictures would show how every mode of interest is affected by the thickness perturbation. Analogous pictures can readily be computed for flat wooden plates, but arched plates present computational problems not yet tackled.

THE BOWED STRING: TOLERANCE RANGES AND WOLF NOTES

We now return to our general consideration of levels of idealization in modelling the violin, and examine what is known about the motion of the bowed string. It is at this point that we first confine our interest strictly to the bowed instruments: our discussion of body behaviour is applicable in general terms to plucked instruments.[33,60] The largest string motion is transverse, and most is known about this; the longitudinal and torsional motions, as well as the effect of bow hair motion, are less well explored.

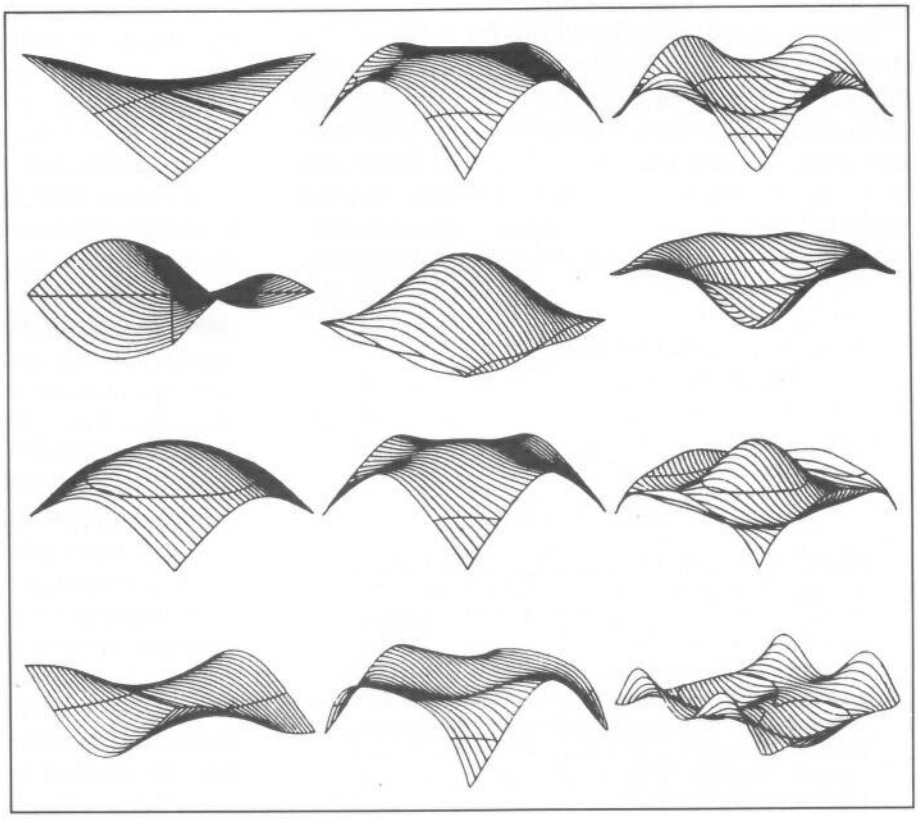

Figure 5. Computer calculations for vibrations of a square, isotropic plate of constant thickness. The first column shows the lowest four vibration modes, with the nodal lines marked. If a small perturbation of thickness is introduced, the change in frequency of each mode is given by the integral of the perturbation with the function shown in column two, while the change in internal damping is given by the integral of the perturbation with the function in column three.

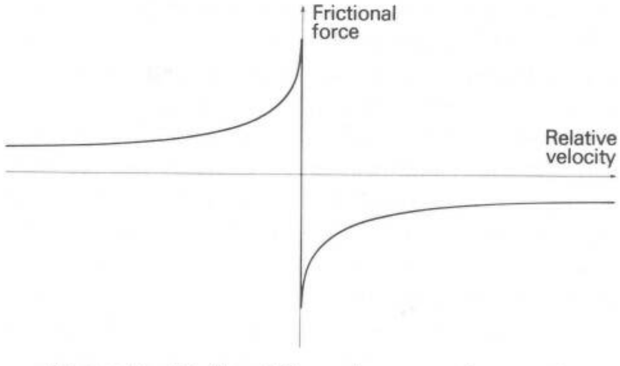

Figure 6. Frictional force between bow and string idealized as a function of their relative velocity, for constant normal bow force.

The transverse vibration of a stretched string has been studied since the earliest days of mathematical physics, and in most textbooks the string is used as the first and simplest example of a continuous vibratory system, that is a system having an infinite series of normal modes of vibration and corresponding natural frequencies. Many scientists are thus surprised to learn that the transverse motion of a bowed string presents many unsolved problems of both practical and mathematical interest. The reason is that the vibrations are excited by the frictional force between bow and string, and this force is a severely nonlinear function of the relative velocity of the bow and string,[5] as illustrated in Fig. 6. This renders the usual mathematical techniques inapplicable. Some of the complexity of the problem is perhaps intuitively apparent if one considers the large variety of sounds elicited from a violin in the hands of a novice, compared with the efforts of a beginner plucking a guitar string.

The motion of a string during the bowing of a steady, *mezzoforte*, musically acceptable note with the bow fairly near the bridge was first observed by Helmholtz.[61] The form of this vibration is somewhat surprising: as illustrated in Fig. 7, at any instant the string is in two approximately straight portions separated by a corner which travels back and forth, tracing out the curved 'envelope' of the vibration

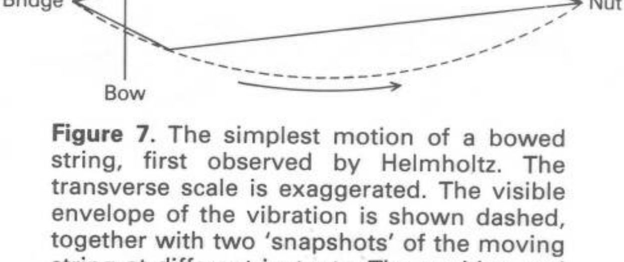

Figure 7. The simplest motion of a bowed string, first observed by Helmholtz. The transverse scale is exaggerated. The visible envelope of the vibration is shown dashed, together with two 'snapshots' of the moving string at different instants. The position and direction of motion of the bow are indicated, and the directions of travel of the Helmholtz corner. The string is sticking to the bow at both instants shown.

visible to the naked eye. Note that the form of this 'Helmholtz motion' is to a large extent independent of the position of the bow, in contrast to the variation with plucking point of the motion of a plucked string.

For most of the cycle the string sticks to the bow, until the 'corner' dislodges it, at which the string flies back rapidly to be recaptured by the bow when the corner arrives back from the bridge. This essential timekeeping role of the Helmholtz corner is what is missing from the usual layman's explanation of the bowing mechanism. A simple but very effective theoretical description of the motion was given by Raman,[21] who worked on violin acoustics in his early years before turning to the work in spectroscopy for which he won a Nobel prize. Raman also studied the hierarchy of possible motions having more than one corner travelling on the string. Such higher types of motion are readily produced when the bow is not too close to the bridge, and they are used by musicians for colouristic effects in *sul tasto* playing.

The usual concern of the player, however, is to set up a Helmholtz motion in the string. He must control three parameters: bow speed, position of the bow on the string, and force between the bow and the string, known perversely by musicians as bow pressure. He must keep these three quantities within certain ranges in order that the steady vibration can exist: these tolerance ranges vary among different violins and are evidently extremely important properties of an instrument. A second and more subtle problem is that he must control the nature and duration of the transient motions of the string, especially the starting transient of the note. The ease with which this can be done may be connected with the steady state tolerance of the instrument, but there is as yet little definite knowledge available.[15,31,62] We discuss mainly the case of a steadily bowed note.

For definiteness, suppose that the position and speed of the bow are kept constant, while the player gradually increases the force from zero. At first he elicits a surface sound in which the fundamental is weak, as in *sul ponticello* playing. He will then find a rather sharply defined minimum bow force at which the Helmholtz motion starts – musicians call this 'getting into the string' – and a less well defined maximum force where the note either goes unacceptably out of tune or gives way to a raucous sound, depending on detailed circumstances.

Minimum bow force was first studied theoretically and experimentally by Raman.[63] Further experimental confirmation of aspects of his theory has been obtained by Saunders[31] and Lazarus.[5] A synthesis and extension of Raman's early work was recently given by Schelleng,[14] in whose penetrating discussion the first explicit theoretical formulae for both bow force limits were given, as well as a good deal of physical insight into the bowed string in general.

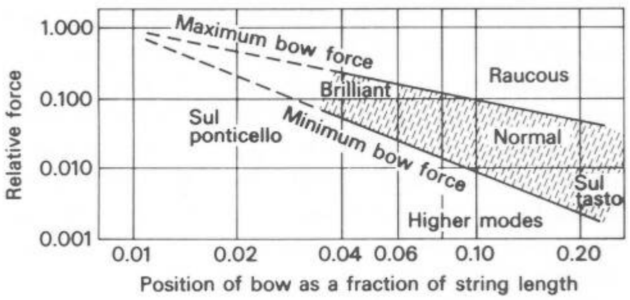

Figure 8. Graph showing the trends of maximum and minimum bow force as the bow is moved away from the bridge, for a given bow speed and a given note on the instrument. The slope of the minimum bow force line is twice that of the maximum bow force line. The graph shows immediately why bowing near the bridge demands greater control on the part of the player. (By permission of J. C. Schelleng and *J. Acoust. Soc. Amer.*)

He summarized the results in a graph of typical force limits against the bowing point as a proportion of the string length, reproduced in Fig. 8. Schelleng's approximate formulae, which in practice should be viewed as giving orders of magnitude only, express the facts that above maximum bow force the arrival of the Helmholtz corner from the nut is insufficient to cause the string to slip, while below minimum force the bow fails to keep hold of the string while the Helmholtz corner is travelling between bow and nut. The surface sound elicited below minimum bow force involves two or more slips per cycle.

We should note that the picture of tolerance ranges given by Fig. 8 becomes over simple when the bowing point moves too far from the bridge. Casual experimentation confirms what Raman's theory predicts, namely that as the bowing point is moved toward a simple fraction such as one-fifth or one-quarter of the string length, the tolerance range fluctuates wildly. The main reason is the appearance of the higher types of vibration already mentioned, which are more complicated than the Helmholtz motion.

It is instructive to look more closely at the mechanism behind minimum bow force. For an ideal textbook string with rigidly fixed ends, minimum bow force would become zero. The Helmholtz motion is a free motion of such a string, so that the bow would not be required at all to maintain it once it had started. In the real situation the bow is needed to sustain the motion against small losses, principally from the ends of the string[5,19] and ultimately attributable in part to radiation losses. As Schelleng made clear, it is the rate of loss of energy from the fundamental that largely determines the position, though not the slope, of the minimum bow force line in Fig. 8, and hence the bowing tolerance range for a given bow position and speed. The losses in turn are largely determined by the behaviour of the instrument body. This appears to be a major reason for variation of tolerance among instruments, and from note to note on one instrument.[63]

The losses tend to be greatest, and hence the bowing tolerance least, at fundamental string frequencies corresponding to resonances of the body.[31,64] Also, at such frequencies unusually large amounts of energy may be stored in body vibrations; these are particularly noticeable to the player in the case of the large peak near the top of the first octave in Fig. 3. This ability to store energy at resonance implies a time lag in the response of the body, which can evidently affect the instrument's transient behaviour.[50] Such a time lag is known to play a crucial role, moreover, in that more bizarre form of intolerance, the 'wolf note'.

THE WOLF NOTE

Suppose that we bow a note whose fundamental frequency coincides with a strong body resonance, and that a Helmholtz motion is set up at the start. The energy stored by the body takes a number of cycles to build up, and during this time there is a continual increase in the rate of energy loss from the string, and hence in minimum bow force, which is dominated by losses at the fundamental, as mentioned before. If the minimum bow force needed exceeds the actual bow force before a steady state is reached, the Helmholtz motion gives way to a 'double slip' regime, whose onset is indicated by the arrows in the top curve of Fig. 9. The energy stored in the body is in just the right phase to promote the growth of this second slip, which takes over as a new Helmholtz motion, out of phase with the old one. Then the whole cycle repeats itself, and the result is the characteristic stuttering sound which is a particularly common problem with certain notes on cellos.

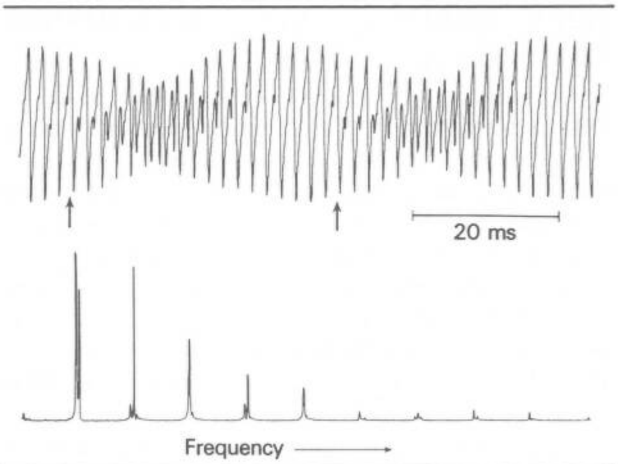

Figure 9. Simple wolf note (played on the G string of a violin). *Top*: waveform of transverse force exerted by string on bridge (measured by the authors using a piezoelectric transducer developed by W. Reinicke). *Bottom*: corresponding frequency spectrum.

This qualitative explanation of the simplest kind of wolf note is substantially due to Raman,[65] although he did not make clear the importance of the phase reversal between alternate cycles of the wolf. This latter point was first brought out by Schelleng,[22] who confirmed Raman's picture using an alternative approach to the simplest wolf note starting from the idea that 'its behaviour immediately suggests beating and coupled circuits'.

In technical language, Schelleng's is a frequency-domain viewpoint, complementing the time-domain view of Raman. As might be expected in a strongly nonlinear problem, some features are easier to understand from one viewpoint and *vice versa*, but the two approaches are consistent.† The frequency-domain viewpoint is suggested by the second curve of Fig. 9, which shows the frequency spectrum of the wolf note in the top curve. The fundamental is split into two peaks of comparable amplitude as predicted by Schelleng, and the wolf phenomenon can be described in terms of beating between these.

Raman's viewpoint readily explains why pressing harder with the bow can suppress the wolf, at least in some cases, while Schelleng's led him to a simple quantitative criterion for susceptibility to wolves which shows why cellos are more prone to them than violins, although Fig. 9, as it happens, comes from a violin. Both approaches explain why wolves can be alleviated by fitting a correctly tuned wolf eliminator[66] or a lighter string, either of which reduces the coupling of the string to the body and hence the maximum stored energy. More complicated wolves, some of which are analogous to so called multiphonics in wind instruments, have been observed by Raman,[65] Firth and Buchanan[67] and others, and have been discussed by Benade.[3] A quantitative theory covering all known cases has yet to be constructed and verified.

MORE REALISTIC MODELS: ROUNDED CORNERS

Real strings on real instruments have slightly anharmonic overtones when allowed to vibrate freely, and losses occur at different rates for different overtones.[3,5,19,68] One result of this is that the Helmholtz corner is not perfectly sharp, but is somewhat rounded. Such corner rounding influences the string's response to the bow in a musically significant way, and models taking account of it are needed. Such models are analogous to those used by Benade and others[3] in work on wind instruments. However, the friction curve nonlinearity is much more ill behaved than the nonlinearities encountered in wind instruments, so Benade's approach poses greater difficulties in the case of the bowed string.

The mathematical technique most promising at first sight is to seek exactly periodic motions, and investigate their stability. Predecessors to this line of work were the studies by Friedlander[69] and Keller,[70] although they used the simplest textbook idealisation of the string. Their models produced some interesting mathematics – indeed, this is the only area of our subject which has so far yielded problems of great formal interest – and Friedlander found the significant result that dissipation is essential for stable, periodic motion.

In the models studied by Raman, Friedlander and Keller there is no change in the string motion as bow force is varied between maximum and minimum. An ideally flexible string with a frequency independent loss mechanism gives no corner rounding, and as Cremer and Lazarus[71,72] first pointed out, corner rounding is necessary for the observed variation of string motion within the tolerance range. We thus require rounded corner models to study such variation. (It is in constructing such models that the input admittance curves mentioned on p. 162 are relevant.)

Such models also make possible a closer study of maximum bow force. As Schelleng realized,[14] maximum bow force in practice is generally less than the force causing catastrophic breakdown of the musical sound to a raucous sound, and it is signalled by one of several less drastic phenomena. Of these phenomena, two matter a great deal to the musician. The first is that the noise content of an otherwise musical note can reach an unacceptable level; we shall return to this subject in the next section.

The second is the very slight deviation of pitch – almost always on the flat side of the string tuning – as bow force increases. This flattening is easily demonstrated with the bow a moderate distance from the bridge at a low bow speed.[73] When flattening is audible, pitch is sensitive to bow force so that control of intonation becomes difficult. Players avoid this regime. It has recently become apparent[74,75] that the flattening effect is strongly connected with the mechanism of waveform change through corner rounding, and so we discuss both phenomena together.

When a rounded Helmholtz corner passes the bow, the effect of the friction curve nonlinearity is to sharpen the corner to an extent which depends very much on bow force. Thus a periodic motion of the string represents an equilibrium between corner rounding by the string and its terminations, and corner sharpening by the bow. As bow force is increased corner sharpening at the bow becomes more pronounced, so that the periodic solution has more high frequency content.[71,72]

A further effect comes into play when bow force exceeds a certain value: the amount of corner sharpening during the transition from sticking to slipping, release, is greater than that during the transition from slipping to sticking, capture.[75] The result of this asymmetry is not only a further change in the shape of the periodic waveform, but also a delay in the round trip time of the Helmholtz corner. In other words, as bow force is increased beyond a certain

†But the time-domain viewpoint penetrates the problem further: see ref. 80, Fig. 14.

limit there is a tendency for the note to play flat, and the degree of flattening increases with bow force as observed.[74–76]

There is yet another consequence of corner sharpening by the bow, first pointed out by Cremer and Lazarus[71,72] and further elucidated by Schelleng.[14] When the shape of the corner transmitted past the bow is changed, a reflected wave is also generated, travelling in the opposite direction. These secondary waves can then reverberate in one section of the string, reflecting from the sticking bow, until they happen to arrive at the bow during a slipping phase in the Helmholtz cycle. The reverberating secondary waves give rise to a characteristic pattern of ripples in the string motion away from the bow, which is almost certainly an important ingredient of tone quality,[77] although the requisite listening tests have yet to be made.

In real strings, ripples are also visible in the waveform of string velocity at the bow: during the sticking part of the Helmholtz cycle, the observed velocity is not exactly equal to the bow velocity.[5,14,71] This is not surprising, since the bow hair can yield slightly; moreover the string can roll on the bow hair. In fact the latter effect is the larger of the two,[5,13,14] and indeed torsional motions are an important aspect of bowed string behaviour altogether. They affect the details of the flattening and corner-sharpening effects already described and they result in extra losses, thus changing the bowing tolerance limits. Since torsional yielding of strings is affected by string diameter and construction while it is not much affected by string tension, it is clear that bowing tolerances will vary with different types of strings, as players are well aware.

A theoretical investigation of the effects of torsion presents no great difficulty in principle: any method capable of analysing transverse motion can rather simply be extended to include torsion.[14,76] The torsional oscillations are known to have a fundamental frequency several times higher than those of the transverse motion,[5,14] but their rate of damping has not yet been satisfactorily measured. More detailed observations and theory will no doubt become available soon.

NOISE, CARRYING POWER, AND OTHER SUBTLETIES

The sound of a real bowed string is more or less noisy, and in the previous section we mentioned that maximum bow force can in some circumstances be governed by the buildup of noise. This is especially apt to happen when trying to play more and more loudly near the bridge. This noisy regime is often used to deliberate musical effect, but the noise can reach an unacceptable level, depending on context.

The presence of noise indicates some kind of departure from the periodic motions discussed so far. Such departures have been observed: Cremer[72] and others have reported measurements of the period lengths of many cycles of a bowed string signal in which variations of up to 30 cents between the shortest and longest cycles were found (a cent is a hundredth part of a semitone). The observation raises two questions. First, is this the source of the audible noise? Psychoacoustical data[78] taken with headphones suggest that such jitter, or random variation of period, need only attain 20 cents or so to be audible. Second, what is the source of the variation? It could be simply a reflection of external factors such as the unequal distribution of rosin on the bow, but on the other hand it could in some way be intrinsic to the bowed string mechanism, as Cremer suggested. This would seriously limit the usefulness of theoretical studies of strictly periodic motion.

The present authors have recently made some careful measurements of jitter. The extremely steep flyback in the sawtooth wave felt by the bridge when a Helmholtz motion is imparted to the string makes possible an accurate determination of period length. One example from our results is shown in Fig. 10: it appears that maximum 'flyback jitter' can be as little as 3 cents for open strings on a real instrument. This observation suggests that the bowed string is indeed capable of precisely periodic motion under at least some playing conditions. On the other hand, different conditions, such as when playing a note high on the violin G string, can produce much more jitter; and in fact the amount of jitter is found to be roughly proportional to the amount of corner rounding, in the sense of the previous section. This is consistent with the idea that unevenness of bow hair and rosin is primarily responsible for jitter.

We should not jump to the conclusion that jitter of less than 20 cents can have no musical effect. Benade[3] (second reference) has suggested that in consequence of the extraordinary sensitivity of the acoustical behaviour of a large concert hall to any slight unsteadiness in the sound source, jitter may reduce the carrying power of an instrument, that is to say its audibility in a concert hall against a background of other sound. The amount of jitter needed to affect carrying power might be smaller than the threshold for direct audibility in headphones.

Our jitter measurements do however suggest that the audible noise is not connected with flyback jitter. Even during loud playing near the bridge, measured flyback jitter often remains well below 20 cents. The source of the audible noise becomes apparent, however, in Fig. 11. This shows transverse force at the bridge during a noisy note, which for an ideal Helmholtz motion would be a sawtooth wave. We note the aperiodic spikes superimposed on this sawtooth: many observations have shown that these always appear when the audible noise builds up.

The main clue about the source of spikes came from experiments in which strings were bowed with a smooth, round, rosined stick in place of a bow. It

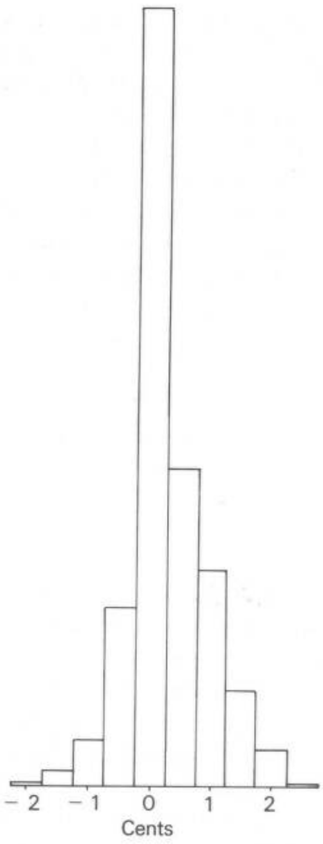

Figure 10. Statistical distribution of period lengths (expressed as departures from a nominal constant value) for 560 cycles of a Helmholtz motion bowed by hand on the open E string of a violin. Special care was taken to minimize spurious effects such as drift in string temperature. The total spread is only three cents or 0.2%.

should be noted, incidentally, that such a stick is much closer than a real bow to most of the theoretical idealizations used so far, and as such may prove an important tool for linking experiments, listening tests and theory in a wider context than the present investigation of spikes. With the stick, spikes are conspicuously absent from the bridge-force waveform; and further experimental and theoretical evidence has recently made it clear that the finite width of the ribbon of bow hair in contact with the string is the essential ingredient in spike production.[74]

This discovery reinforces a point made by Schumacher,[13] that we should examine more closely the influence of the bow itself. The bow hair and the stick to which it is attached are by no means rigid, and will yield to some extent under the fluctuating friction force. Indeed, we would expect to find some significant effect of bow dynamics on string motion since players can distinguish between different bows by their playing properties.

Investigation into bow behaviour has been relatively neglected in the past. Measurements on various bows have been made by Schumacher,[13] who has also taken the first steps toward incorporating bow hair motion in a theoretical treatment of string

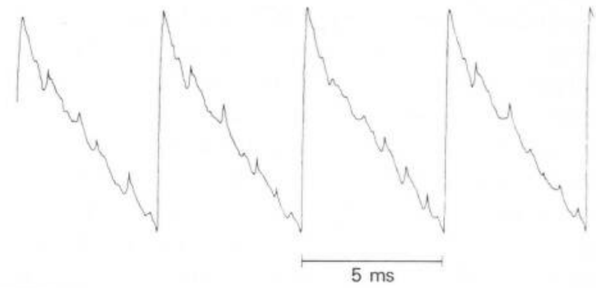

Figure 11. Waveform of transverse force exerted by string on bridge for a loudly bowed open violin G. The main 'flybacks' occur when the Helmholtz corner reflects from the bridge, and their timing is extremely regular. By contrast, the 'spikes' in between are aperiodic, and cause audible noise. (Measured by the authors using a piezoelectric transducer developed by W. Reinicke.)

motion.[76] The mathematical problem is closely analogous to that of incorporating torsional motion of the string, since again the extra equation involved is linear, and couples to the others only through the friction force. This is only the case, however, as long as the bow is idealized as having infinitesimal width. To allow fully for finite width of the bow and thus simulate spike production would be a much more complicated undertaking.†

The final complication is longitudinal string motion.[3,15,48] Since the length of a string is increased somewhat when it vibrates transversely, its tension will also increase, so that at the bridge the fluctuating force will have a component along the string as well as transverse to it. A component of this will be exerted downward on the bridge, and hence on the instrument. The waveform of this 'indirect excitation' will depend on the behaviour of the longitudinal waves in the string; their fundamental frequency is typically about three octaves above the fundamental transverse frequency, as is well known to players who inadvertently excite them when cleaning rosin from their strings.

The coupling of these waves to the transverse motion is nonlinear;[17] moreover one finds that transverse motion with sharp corners in it, like the Helmholtz motion, is particularly effective for exciting longitudinal motion. Also, the nonlinear coupling implies that a feature of the transverse string displacement which we omitted for clarity from Fig. 7 becomes important. There is a stationary corner on the string at the bow caused by the friction force there; and this force has a substantial steady component. The result is a significant indirect excitation at the fundamental as well as the octave and higher harmonics, a point which distinguishes the violin from the guitar and which has sometimes been overlooked.

It appears certain that longitudinal motion is excited in a bowed string, and may have audible

†Progress has nevertheless been made (ref. 80, p. 1343 & refs).

consequences[15] – especially in starting transients, and especially with metal strings which have a higher Young's modulus than gut strings. The strength of the indirect excitation is obviously sensitive to the angle the strings make with the bridge, a parameter long regarded by makers as critical. A promising first step toward assessing the importance of indirect excitation would be to combine Mathews' and Kohut's technique with a simple computer simulation of the longitudinal string motion, to give the possibility of a listening test in which the indirect excitation could be switched in and out.

In another ten years, a review similar to this should be able to report very substantial progress towards understanding a number of features of stringed instrument behaviour which are of real concern to musicians. As has been made clear here, many lines of investigation are now being pursued which promise to move toward this goal, many of which have only recently become available with the development of computer technology to its present level.

Neither violin makers nor violin players will be displaced by this knowledge, but possibly the world's very limited supply of top class instruments will eventually be augmented when a better understanding of what makes a top class instrument, and top class bows and strings, enables them to be produced slightly more reliably. Even more important, such knowledge might raise the standard of mass produced students' instruments from the present very low level to something at least decent, if not excellent. If in the process our understanding of auditory perception mechanisms also develops, as was suggested in the introduction, that will be a valuable bonus.

Acknowledgements

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Note that some of the papers referred to below, as well as many others relating to violin acoustics, are reprinted in *Musical Acoustics*, Parts I and II, ed. by C. M. Hutchins (Benchmark Papers in Acoustics **5**, **6**), Dowden, Hutchinson and Ross, distributed by Wiley-Halsted, New York and London (1975/1976). The relevant papers are marked * (Part I) and † (Part II) below.

Much of the most recent work is as yet reported only in the *Catgut Acoustical Society Newsletter*. Readers having no access to this may write to: 112 Essex Avenue, Montclair, N.J. 07042, U.S.A.

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