(g) Under the transformation $Q \to -Q; f \to -f$, the implied flow on the slow manifold, $\{\boldsymbol{u}_s, h_s\}$, will remain on the slow manifold, and be transformed according to $\boldsymbol{u}_s \to -\boldsymbol{u}_s; h_s \to h_s$.

This last property was termed "sign consistency" by McIntyre & Norton (1993), and it is a property of all standard balanced models, such as the quasigeostrophic and semigeostrophic equations. Property *(g)*, where it applies, is a strong form of property *(e)*, and it is equivalent to giving the same force to property *(e)* as that which was given to property *(d)*: namely, that under the relevant transformation, the slow manifold is not simply mapped onto any slow manifold, but it is mapped onto the same slow manifold. If $\mathcal{M}$ were assumed to be unique, then property *(g)* and property *(e)* would be equivalent.

It will now be shown that, for the shallow water equations at least, a slow manifold possessing properties *(a)* – *(g)* cannot exist. This is done by first supposing that there is such a slow manifold, $\mathcal{M}$, say, and then deriving a contradiction.

Firstly, recall the vortex instability result. In an unbounded domain, it was shown that the vortex was temporally unstable for all Froude and Rossby numbers. Moreover, it possesses a unique temporally growing mode for each azimuthal mode number m, provided m is sufficiently large, and the amplitude of the mode is assumed to be finite at infinity. It also possesses a unique temporally decaying mode for each such m, assuming that the amplitude of the mode is finite at infinity. We shall fix attention on a single such azimuthal mode number, m, and denote the growing and decaying eigenmodes for that mode number by $\phi_\uparrow(r)$ and $\phi_\downarrow(r)$ respectively. Here, the functions $\phi_\uparrow(r)$ and $\phi_\downarrow(r)$ are taken to represent the three-vector (u, v, h) in each case. In the limit $Ro \to 0$, $\phi_\uparrow$ and $\phi_\downarrow$ coincide, since the growth rate of the instability is of small order in Rossby number.

Now, let us consider the nature of the evolution of the vortex with a perturbed boundary, under the assumption of linearized perturbations. The boundary of the vortex has and undulation on it, of small amplitude, and of fixed mode number m. On the slow manifold $\mathcal{M}$, it follows from property *(f)* and linearization that, at any instant in time, the amplitude and phase of the vortex boundary undulation are sufficient to determine the perturbation velocity and height fields everywhere. Since we are considering linearized perturbations, we can represent the radial

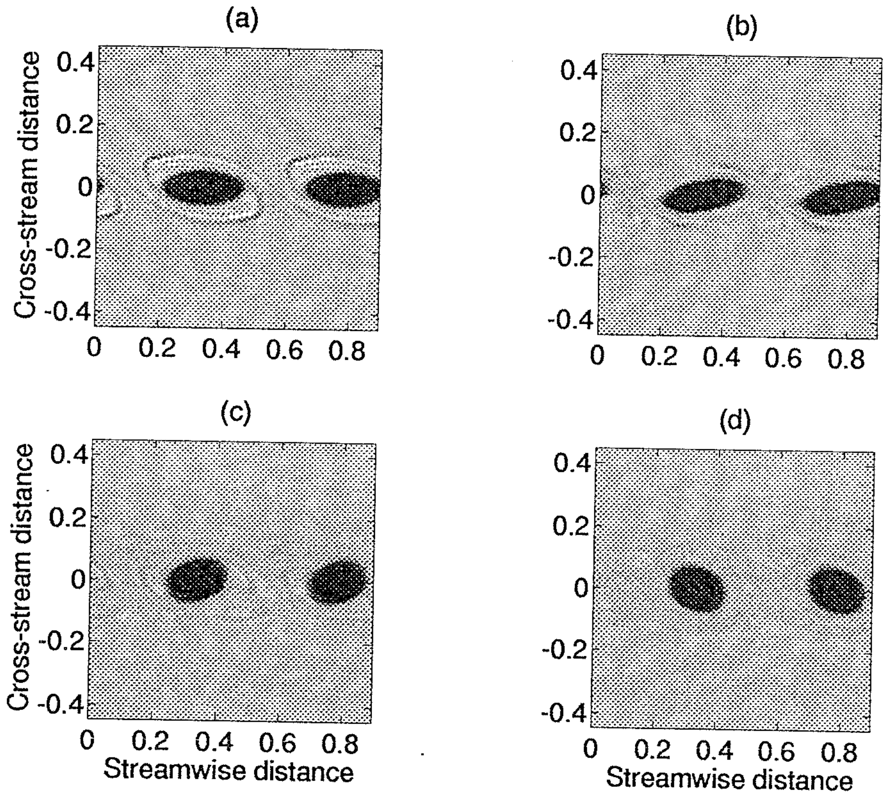

Figure 4.6: Nonlinear evolution of potential vorticity field, simulation **A**ii. Figures a,b,c,d correspond to times 6.9, 15.0, 23.1 and 31.2 respectively. Two periods of the model domain are shown.