Equatorial Waves and Tropical Dynamics

Tropical Meteorology

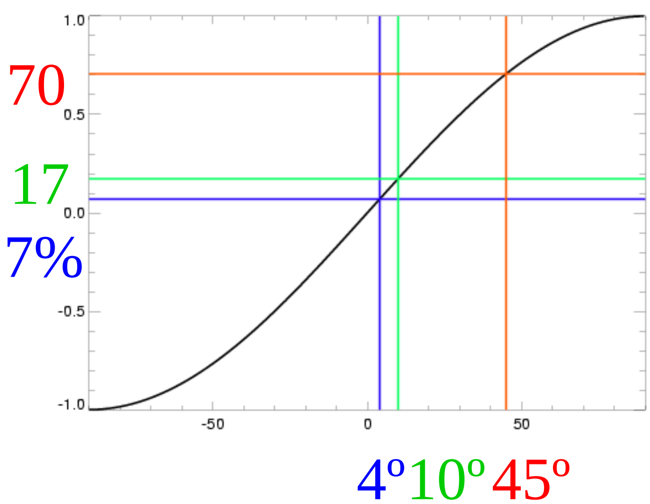
Dynamics of equatorial regions very different to midlatitudes. Low latitudes can influence higher latitudes through ‘teleconnections’. E.g.: influence of the El Nino Southern Oscillation (ENSO)

Major difference midlatiudes vs tropics:

Coriolis $f = 2\Omega \sin\varphi$

small in tropics

$\beta = \partial f / \partial y$ largest at equator



Earth is spherical and rotates!

Energy sources

1. lateral forcing from midlatitudes

2. barotropic energy conversions

3. systematic interaction with convective heating

Governing equations

In log-pressure co-ordinates ($z^* \equiv -H \ln \frac{p}{p_0}$, $w^* = \frac{Dz^*}{Dt}$

with $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{u}_\mathrm{h}.\nabla + w^* \frac{\partial}{\partial z^*}$):

Momentum:

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_h \bullet \nabla + \omega^* \frac{\partial}{\partial z^*}\right)\mathbf{u}_h + f\mathbf{k} \wedge \mathbf{u}_h = -\nabla\Phi \qquad (4.1)$$

Hydrostatic: $$\frac{\partial \Phi}{\partial z^*} = \frac{RT}{H} \qquad (4.2)$$

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w^*}{\partial z^*} - \frac{w^*}{H} = 0 \qquad (4.3)$$

Thermodynamic:

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_h \bullet \nabla\right)T + \frac{w^* N^2 H}{R} = \frac{J}{c_p} \qquad (4.4)$$

Scaling analysis

$D \sim 2.5\times10^3\,\mathrm{m}$	vertical depth
$L \sim 10^6\,\mathrm{m}$	horizontal length
$U \sim 5\,\mathrm{ms}^{-1}$	horizontal velocity
$\tau \sim 10^5\,\mathrm{s}$	time
$H \sim 8\times10^3\,\mathrm{m}$	scale height
$N \sim 1.2\times10^{-2}\,\mathrm{s}^{-1}$	buoyancy frequency

$$\frac{\partial \mathbf{u}_h}{\partial t} + \mathbf{u}_h \bullet \nabla \mathbf{u}_h + \omega^* \frac{\partial}{\partial z^*}\mathbf{u}_h + f\mathbf{k} \wedge \mathbf{u}_h = -\nabla \Phi$$

$$\frac{U}{\tau} \qquad \frac{U^2}{L} \qquad \frac{WU}{D} \qquad fU \qquad \frac{\delta\Phi}{L}$$

Midlatitudes: $f \sim 10^{-4}\,\mathrm{s}^{-1}$ and $\mathrm{Ro} \sim U/fL$ small so Coriolis term balances pressure gradient term.

Tropics: $f \le 10^{-5}\,\mathrm{s}^{-1}$ and $\mathrm{Ro} \ge 1$. If assume vertical velocity W small, Coriolis term $fU \sim 10^{-4}\,\mathrm{ms}^{-2}$ usually dominant on LHS, hence $\delta\Phi \sim 100\,\mathrm{m}^2\mathrm{s}^{-2}$.

From hydrostatic equation $T \sim \dfrac{H}{R}\dfrac{\delta\Phi}{D} \sim 1\,\mathrm{K}$.

Small size of f in tropics: leads to small geopotential (hence temperature) fluctuations vs midlatitudes.

Temperature fluctuations associated with available potential energy:

- in tropics disturbances must grow by means other than baroclinic energy conversions.

Consider thermodynamic equation & assume flow adiabatic (J=0)

$$\frac{\partial T}{\partial t} + \mathbf{u}_h \bullet \nabla T + \frac{w^* N^2 H}{R} = 0$$

$$\frac{T}{\tau} \qquad\qquad \frac{UT}{L} \qquad\qquad \frac{WN^2 H}{R}$$

Local time rate of change of temperature dominates horizontal advection and hence

$W \sim \dfrac{RT}{HN^2\tau} \sim 2.5\times10^{-3}\mathrm{ms}^{-1}$ – vertical velocity small as we assumed.

Balances in continuity equation:

$\dfrac{\partial u}{\partial x} \sim \dfrac{\partial v}{\partial y} \sim \dfrac{U}{L} \sim 5\times10^{-6}\mathrm{s}^{-1}$, $\dfrac{\partial w^*}{\partial z^*} \sim \dfrac{W}{D} \sim 10^{-6}\mathrm{s}^{-1}$ and

$\dfrac{W}{H} \sim 3\times10^{-7}\mathrm{s}^{-1}$. Considerable cancellation between

divergence terms $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} \sim \dfrac{W}{D} \sim \dfrac{\delta\Phi}{N^2\tau D^2}$.

Clouds

Latent heat dominant source of energy of tropical circulation with radiative heating secondary role.

Small scales help determine strength/location of convection that drives large-scale flow.

Distribution of radiative heating/cooling strongly influenced by distribution & physical characteristics of clouds. Parameterised in models.

Equatorial waves

Equatorial waves are trapped near equator. Propagate in zonal vertical directions. **Coriolis force changes sign at the equator**.

Diabatic heating by organised tropical convection can excite atmospheric equatorial waves, wind stresses can excite oceanic equatorial waves.

Atmospheric equatorial wave propagation - effects of convective storms communicated large longitudinal distances - remote response to localised heat source.

Oceanic equatorial wave propagation, can cause local wind stress anomalies to remotely influence thermocline depth and SST.

Fastest moving waves are eastward propagating **Kelvin waves**. Typical Kelvin waves for Pacific move at ~3 ms^{-1} and take ~2 months to cross Pacific.

Other waves: **equatorial Rossby waves**. More slowly moving (fastest Rossby wave ~1/3 speed of Kelvin wave) & phase velocities propagate westward.

Generation of Kelvin & Rossby waves gives rise to Pacific climate oscillation **El Nino Southern Oscillation (ENSO)**.

Equatorial β-plane

Following Matsuno (1966).

Use shallow water model. Equatorial β-plane ($\cos\varphi \approx 1$, $\sin\varphi \approx \varphi = y/a$, $f \approx \beta y$).

Linearised shallow water equations for perturbations on a motionless basic state of mean depth h_e are:

$$\frac{\partial u'}{\partial t} - \beta y v' = -\frac{\partial \Phi'}{\partial x} \qquad (4.5)$$

$$\frac{\partial v'}{\partial t} + \beta y u' = -\frac{\partial \Phi'}{\partial y} \qquad (4.6)$$

$$\frac{\partial \Phi'}{\partial t} + g h_e \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) = 0 \qquad (4.7)$$

where $\Phi' = gh'$ is the geopotential disturbance.

Seek solutions in form of zonally propagating waves, i.e. assume wavelike solutions but retain y-variation :

$$(u', v', \Phi') = \mathrm{Re}\left\{\left[\hat{u}(y), \hat{v}(y), \hat{\Phi}(y)\right]\right\}\exp[i(kx - \omega t)]$$

Substituting into (4.5-7) gives:

$$\frac{d^2\hat{v}}{dy^2} + \left(\frac{\omega^2}{gh_e} - k^2 - \frac{k\beta}{\omega} - \frac{\beta^2 y^2}{gh_e}\right)\hat{v} = 0$$

Require $\hat{v}$ to decay to zero at large $|y|$.

Schrödinger equation with simple harmonic potential energy. One solution $\hat{v} = 0$. Other solutions exist only for given k if ω takes particular value.

Non-dimensionalise and set

$$\lambda = \left(\frac{\omega^2}{gh_e} - k^2 - \frac{k\beta}{\omega}\right)\frac{\sqrt{gh_e}}{2\beta} - \frac{1}{2}$$ and $\hat{v} = F(Y)\mathrm{e}^{-Y^2/2}$

where $Y = \dfrac{\beta^{1/2}}{(gh_e)^{1/4}}y$

Then can be re-written as **Hermite differential equation** $F'' - 2YF' + 2\lambda F = 0$.

Solutions which satisfy the boundary conditions are $F = cH_{\lambda}$, where $\lambda = n$ for $n = 0,1,2...$ and $H_n(Y)$ is a Hermite polynomial.

The first few Hermite polynomials are:

$H_0 = 1$, $H_1(Y) = 2Y$, $H_2(Y) = 4Y^2 - 2$,
$H_3(Y) = 8Y^3 - 12Y$.

$H_0 = 1; \quad H_1 = 2x; \quad H_2 = 4x^2 - 2; \quad H_3 = 8x^3 - 12x; \quad H_4 = 16x^4 - 48x^2 + 12$

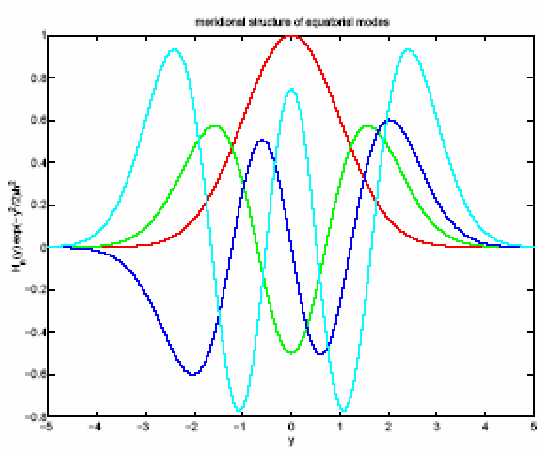


Figure 19: The latitudinal structure of the first few equatorial modes: $H_n(y)\exp(-y^2/2)/n^2$.

Clearly the solutions are 'trapped' near the equator by the exponential.

Horizontal dispersion relation:

$$\frac{\sqrt{gh_e}}{\beta}\left(\frac{\omega^2}{gh_e}-k^2-\frac{k\beta}{\omega}\right)=2n+1,\ n=0,1,2... \qquad (4.8)$$

Cubic in ω, so 3 roots for ω when n & k are specified.

At low frequencies ω, ω^2/gh_e smaller than other terms

& $\omega_{\text{Rossby}} \approx \dfrac{-\beta k}{k^2+(2n+1)\beta/\sqrt{gh_e}}$. **Equatorial Rossby waves**: westward-propagating as ω opposite sign to k.

At high frequencies ω, $-k\beta/\omega$ small &

$\omega_{IG} \approx \pm\sqrt{(2n+1)\beta\sqrt{gh_e}+k^2gh_e}$. Eastward & westward propagating **inertio-gravity waves**.

For case $n=0$, from (4.8),

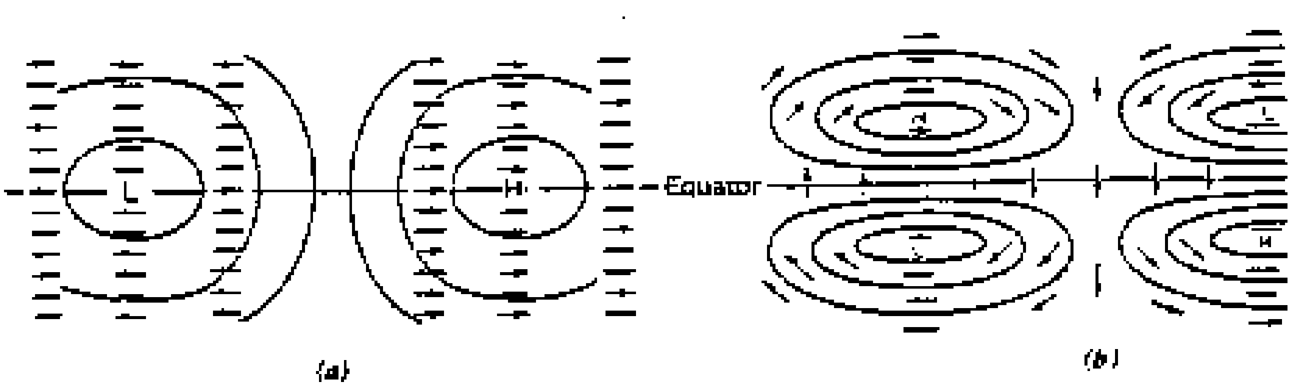
$$\omega_{n=0}=k\sqrt{gh_e}\left(\frac{1}{2}\pm\sqrt{1+\frac{4\beta}{k^2\sqrt{gh_e}}}\right).$$

Eastward-propagating **inertio-gravity** wave &

westward **Rossby-gravity** wave (Yanai wave).

Properties of Rossby & gravity waves in different limits.

For case $\hat{v} = 0$, equations for $\hat{u}, \hat{v}, \hat{\Phi}$ derived from (4.5-4.7) give dispersion relation for fast-moving eastward propagating **Kelvin wave**: $\omega_{\text{Kelvin}} = \sqrt{gh_e}\,k$ with meridional structure of $\hat{u} = u_0 \exp\left(-\frac{\beta y^2}{2\sqrt{gh_e}}\right)$. Zonal velocity and geopotential perturbations vary in latitude as Gaussian functions centred on the equator.



Velocity & pressure distributions in horizontal plane for (a) Kelvin waves, and (b) Rossby-gravity waves (Matsuno 1966).

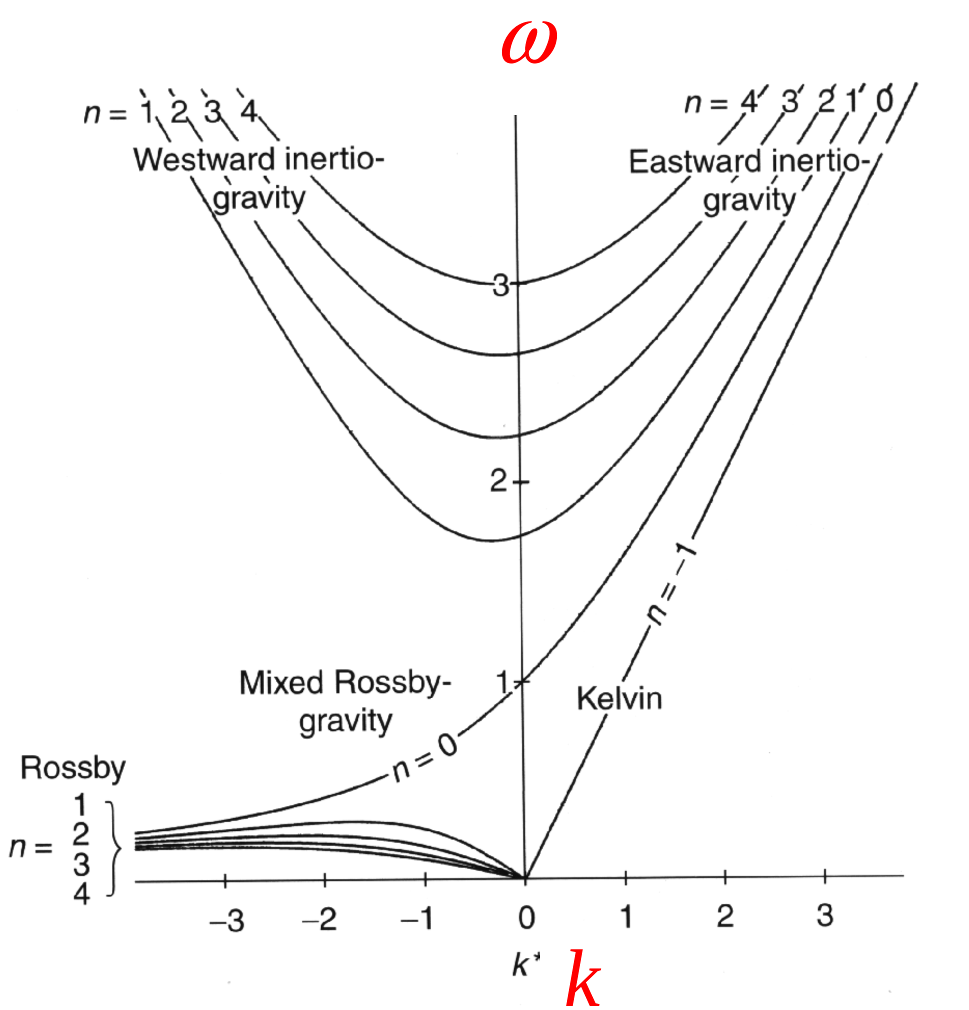


Figure 1 Dispersion curves for equatorial waves (up to $n = 4$) as a function of the nondimensional frequency, ν^*, and zonal wavenumber, k^*, where $\nu^* \equiv \nu/(\beta\sqrt{gh_l})^{1/2}$, and $k^* \equiv k(\sqrt{gh_l}/\beta)^{1/2}$. For all but the Kelvin wave, these dispersion curves are solutions of eqn [13]. Eastward-propagating waves (relative to the zero basic state employed) appear in the right-hand quadrant, and westward propagating waves appear on the left. (Adapted from Matsuno (1966).)

Zonal phase speed $c_p = \omega / k$ (position wrt origin).

Zonal component of group velocity $c_g = \partial\omega / \partial k$ (local slope of curves).

Zoology of equatorial waves

Rossby waves only propagate to west, whereas their energy (group velocity) may propagate to east or west.

Mixed Rossby-gravity waves have westward and eastward energy propagation.

Kelvin waves are non-dispersive with phase propagating relatively quickly to east at same speed as their group.

Eastward inertio-gravity waves have phase and group velocities to east.

Westward inertio-gravity waves have phase velocity to west and group velocity also to west except for very low zonal wavenumbers.

Phase speed

Inertio-gravity > Kelvin > Rossby waves

Typical values of the Kelvin wave speed are in range $c_e \equiv \sqrt{gh_e} \approx 10-50\text{ms}^{-1}$ in troposphere (corresponding to $h_e = 10-250\text{m}$) with higher values in middle atmosphere. For internal ocean waves that propagate along thermocline $c_e \approx 0.5-3\text{ms}^{-1}$ (corresponding to $h_e = 0.025-1\text{m}$).

Scale

The horizontal scale of waves is determined by equatorial Rossby radius, $L = \left(\sqrt{gh_e}/\beta\right)^{1/2}$. For troposphere, with value of h_e above, this gives 6-13º latitude. For internal modes in ocean, it gives 1.3-3.3º latitude.

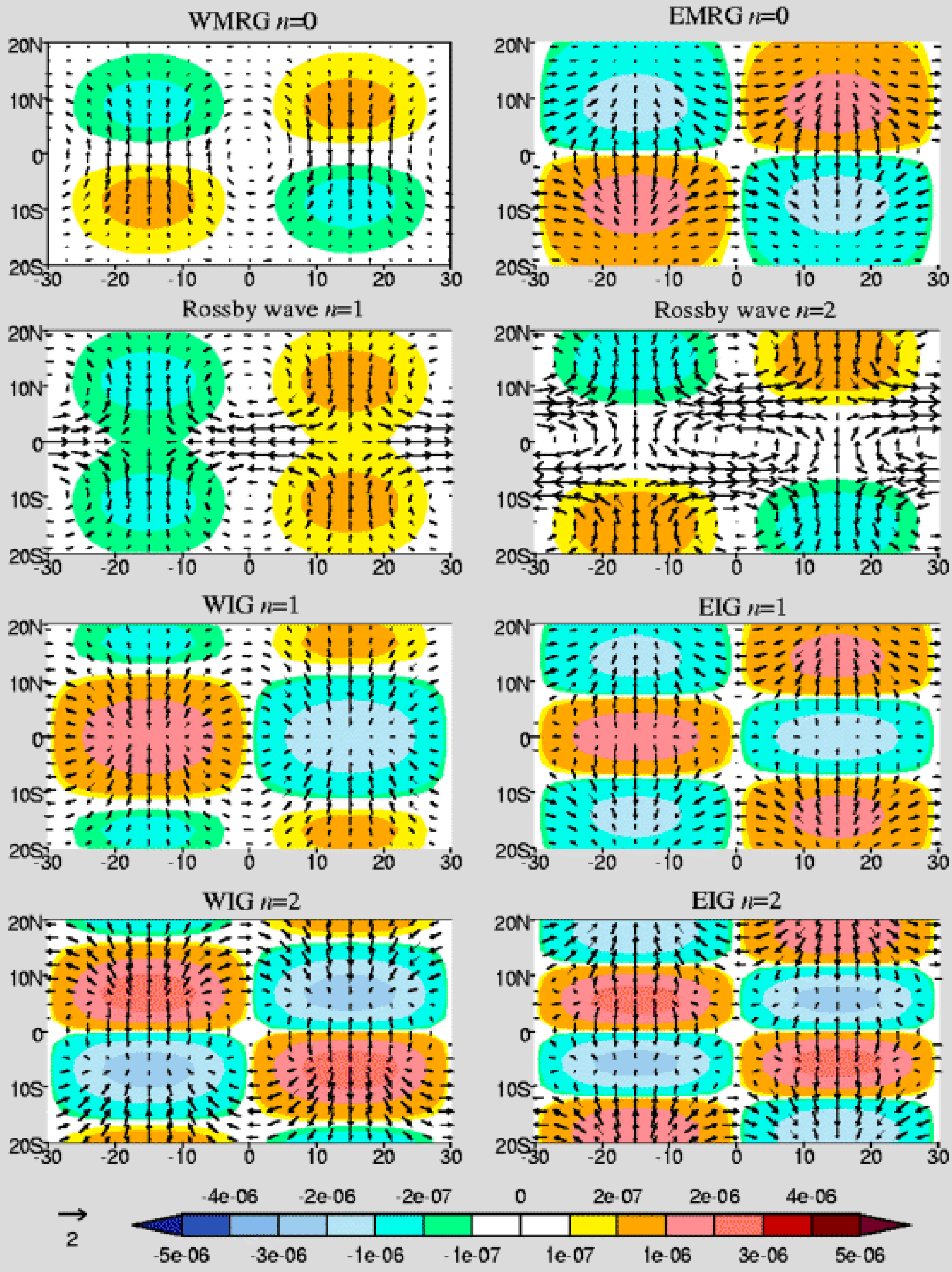
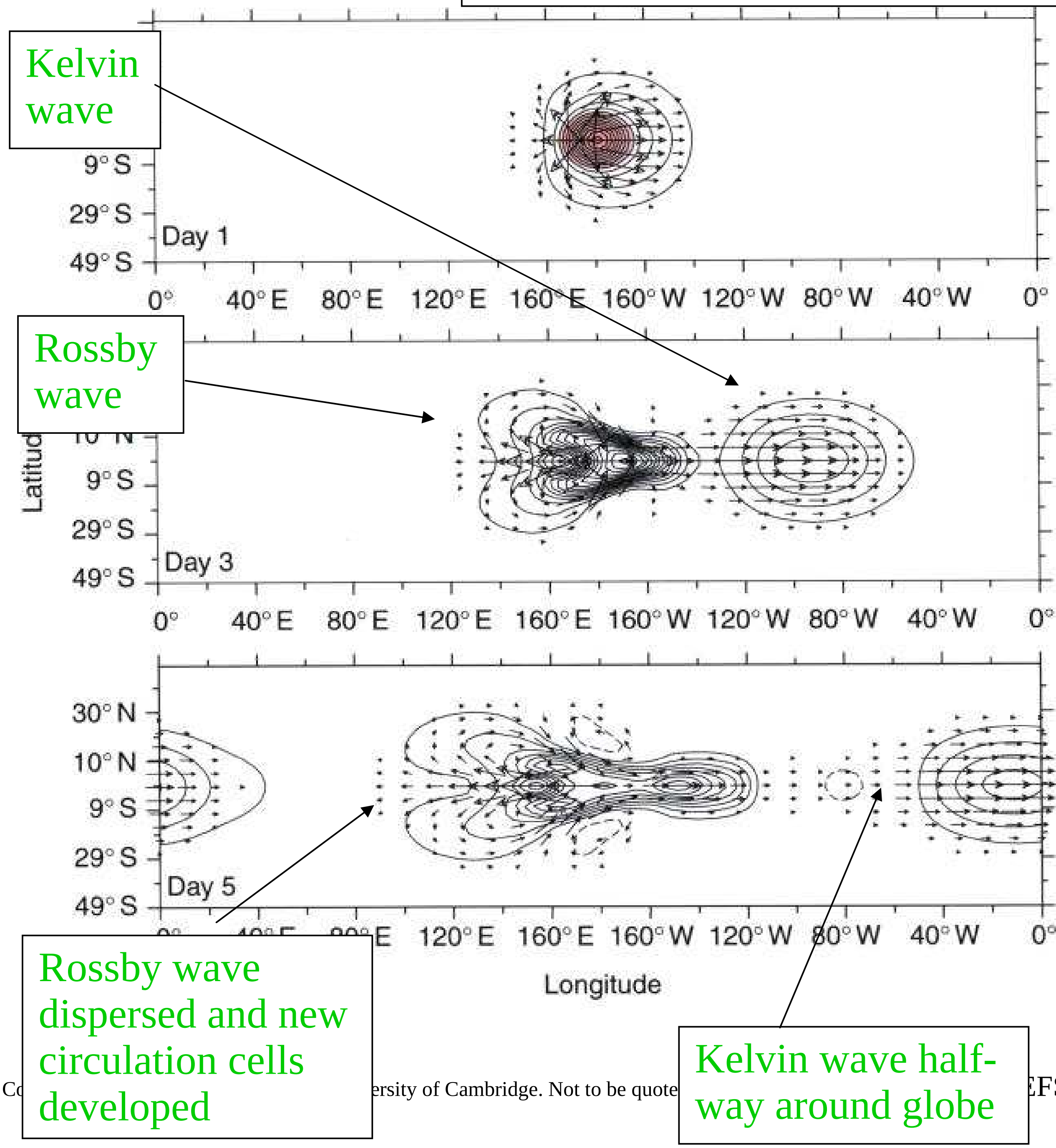


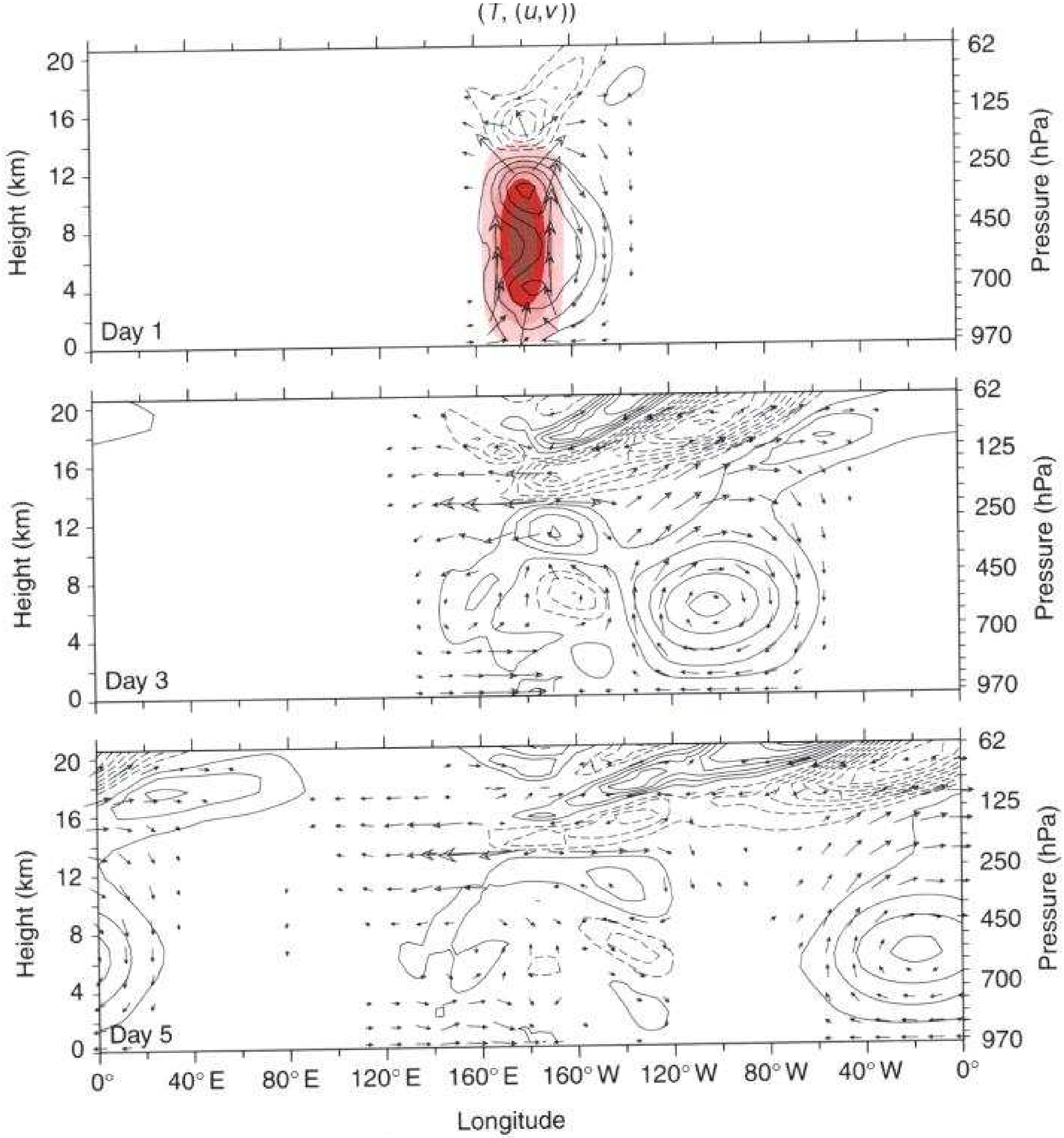
Figure 70. Horizontal wind (vector) and divergence (colour) solutions of theoretical equatorially trapped waves with meridional wind $v_n=D_n(y/y_D)\,e^{i(kx-\omega t)}$. Here D_n is a parabolic cylinder function, n is the meridional mode number, and the trapping scale $y_D=6^{\circ}$ and zonal wave number k=6. The waves shown are the westward mixed Rossby-gravity (WMRG) wave, the eastward mixed Rossby-gravity (EMRG) wave, the Rossby wave, and the westward inertio-gravity (WIG) and the eastward inertio-gravity (EIG) waves. Units are ms^{-1} for winds and s^{-1} for divergence

Model experiment

Multilevel primitive equation atmospheric model forced by imposed heating over 2 days. Heating representative of latent heating in organized convection.

205hPa. Horiz winds (vectors), temp perturbation (contours). Red colour in first panel shows imposed heating

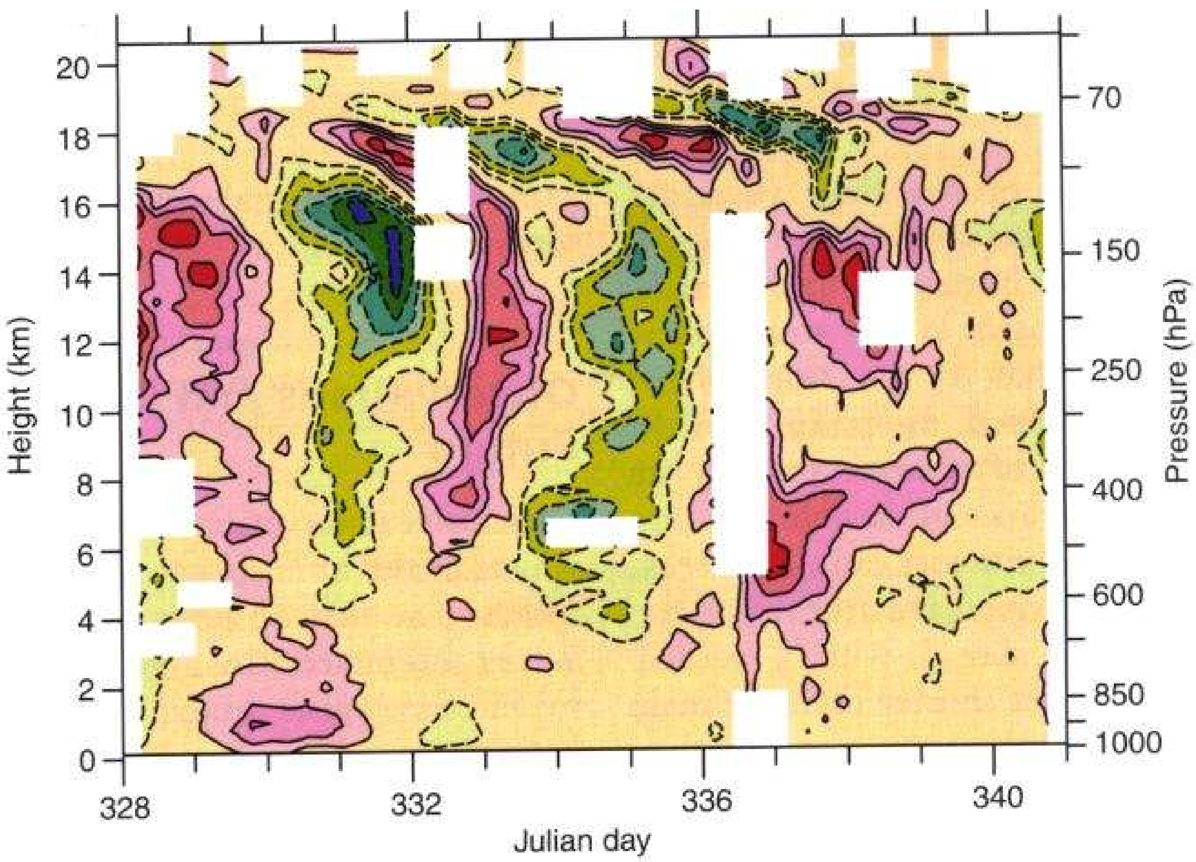




As before but vertical section at equator

Observations

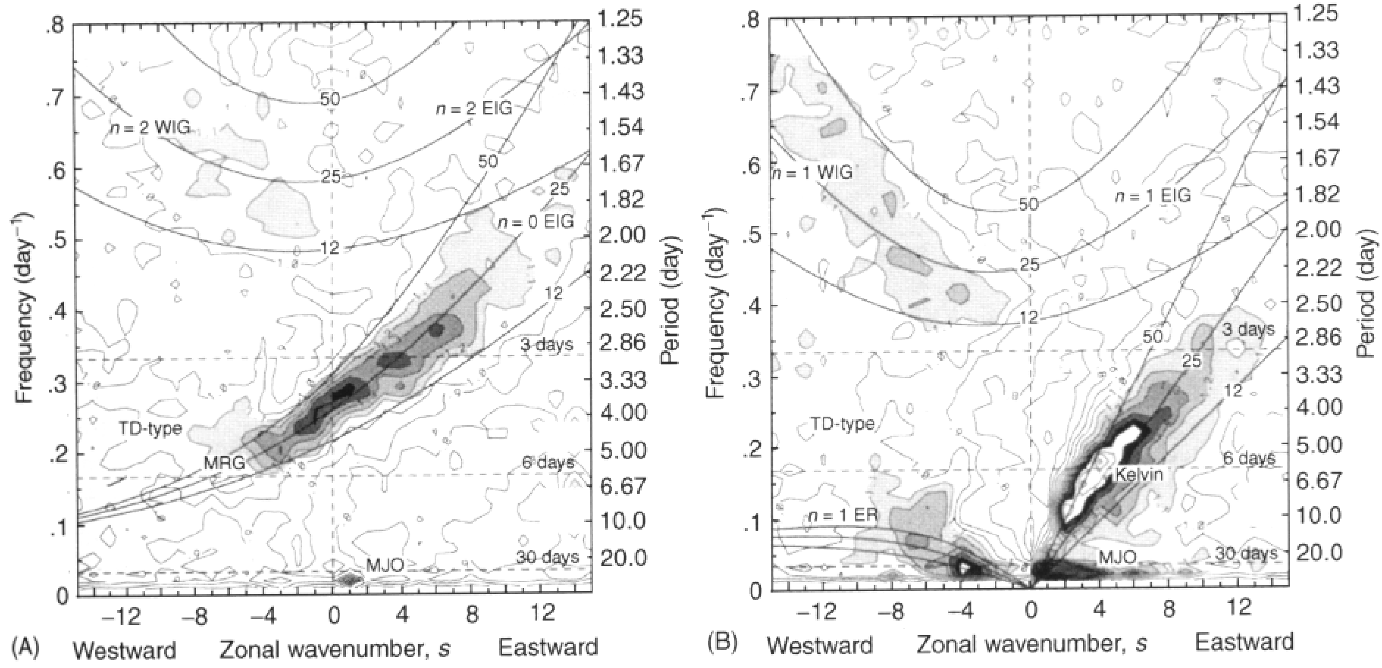
Kelvin waves & Rossby-gravity waves observed from balloon soundings over Pacific. Propagating vertically into stratosphere from tropospheric source (important source of momentum).



Time-height plot of meridional component of wind recorded by balloon soundings taken at equatorial island of Nauru. Contours 3m/s, southerly (red), northerly (green)

Oscillation 4-5 days: Rossby-gravity wave.

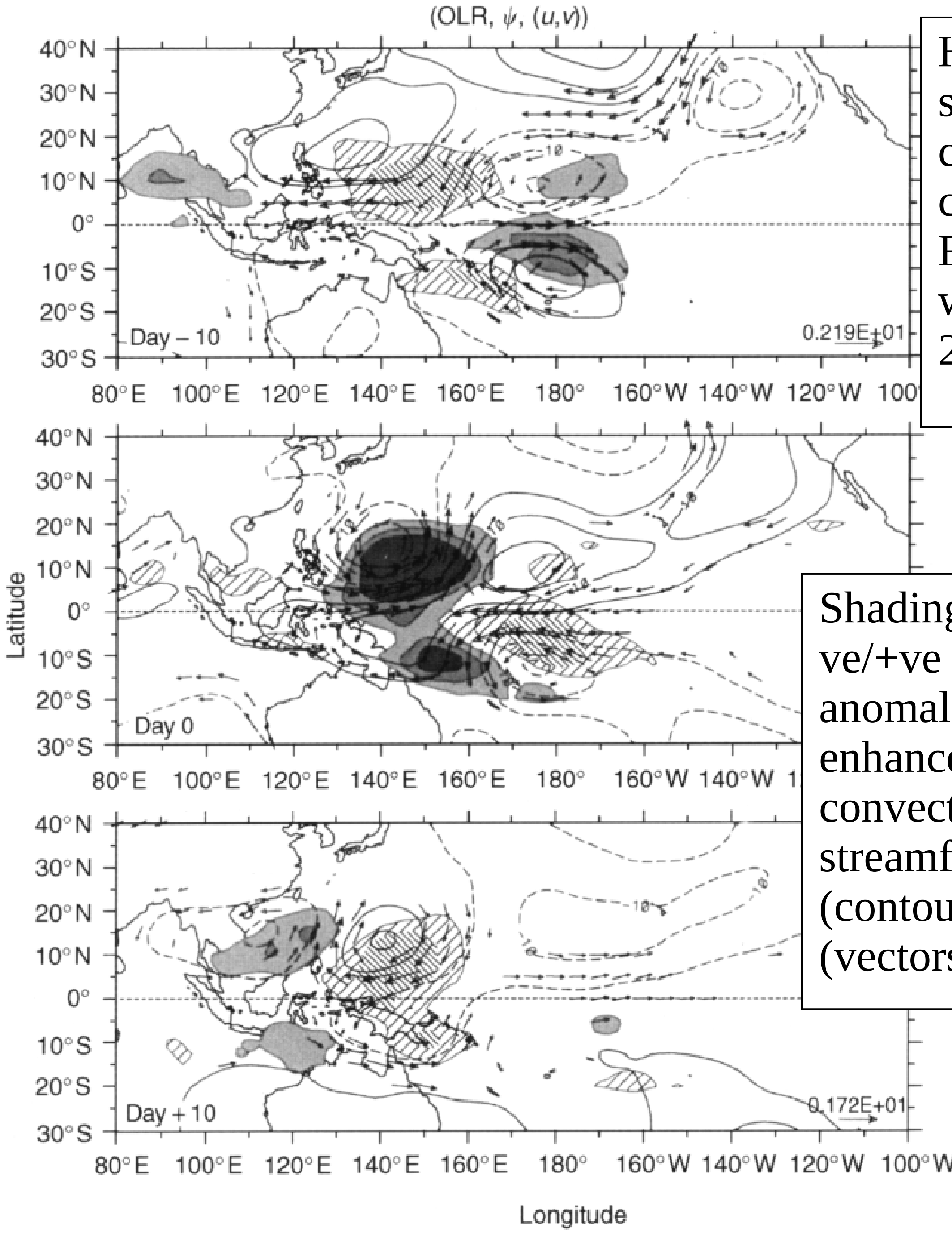
Wave-number Frequency Spectrum of Convectively-Coupled Equatorial Waves



Wave-number frequency spectral peaks of satellite-observed outgoing long-wave radiation (OLR) between 15ºN and 15ºS.

(A) antisymmetric component w.r.t. equator; (B) symmetric component. Superimposed are dispersion curves of equatorial waves (Wheeler and Kiladis, 1999; Wheeler et al, 2000).

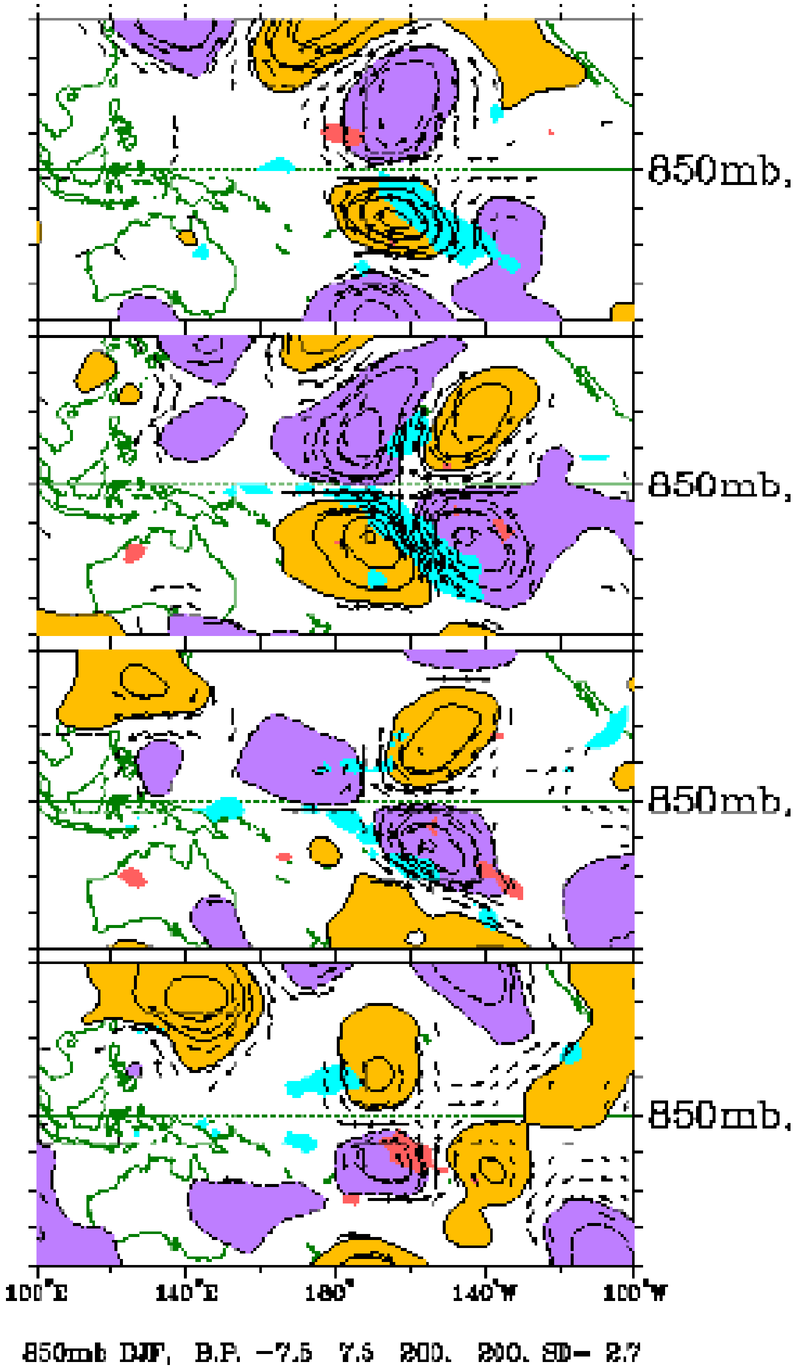
Equatorial Rossby Wave n=1



Horiz structure of convectively coupled n=1 Rossby wave over 21 days.

Shading/hatching = -ve/+ve OLR anomalies, enhanced/suppressed convection. 850hPa streamfunction (contour) & wind (vectors) anomalies.

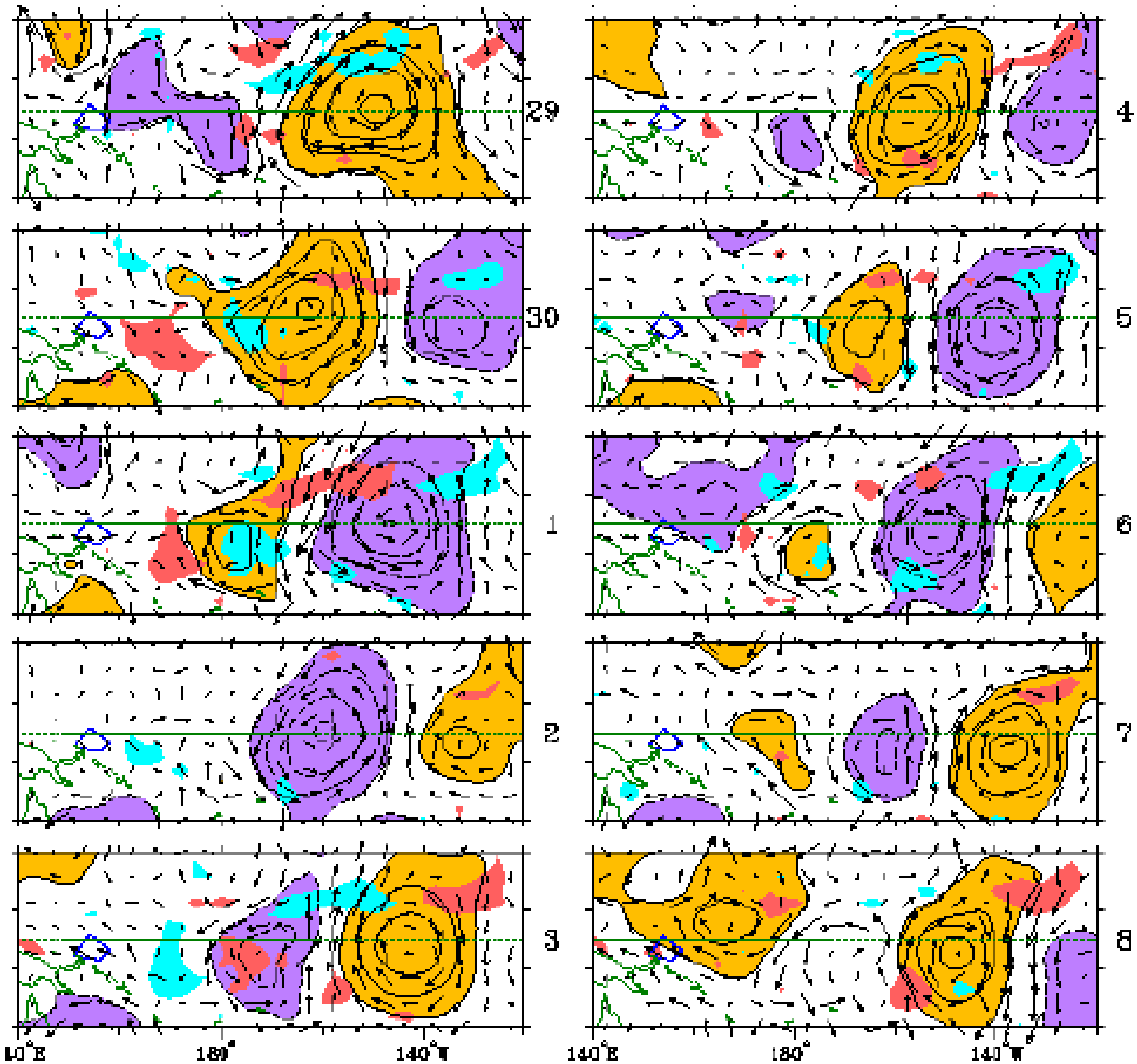
Typical lower tropospheric Rossby wave. Suppressed convection in conjunction with equatorward flow. Modulates large-scale tropical weather.



850mb streamfunction contours with orange/purple shading + wind vectors and OLR anomalies - suppressed (red) & enhanced (blue) convection of an observed n = 1 equatorial Rossby wave.

Period of disturbance ~12 days. Enhanced convection in poleward flow, suppressed convection in equatorward flow (Kiladis and Wheeler, 1995)

Mixed Rossby-gravity wave



Westward phase speed of waves, period ~5days. Asymmetric convection patterns. The latitudinal coverage of these plots 17.5°N to 17.5°S. (TOGA-COARE)

