Troposphere/Stratosphere Interaction

A Simple Model

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Overview

• Observations of zonal deceleration in the Stratosphere and the Wave Response in the Troposphere

• Explanation of the Simple Model

• Analytical Solutions show Tropospheric response to descending Stratospheric deceleration.
Descent of Stratospheric Deceleration

The Vertical Structures of the EOFs

Winter EOFs from NOAA-CIRES Climate Diagnostic Center, Boulder CO.
Zonal Mean is seen in the Stratosphere
Wave Reaction is seen in the Troposphere
1985

Zonal

Wave 1

Wave 2

60° N
The Quasi-Geostrophic Equations

Linearized Potential Vorticity
\[
\epsilon(u_0 - c)(\Psi_{zz} - \Psi_z) + (u_0 - c)\Psi_{yy} + \Psi[-k^2(u_0 - c) - \epsilon(u_{0zz} - u_{0z}) + \beta] = 0
\]

Linearized Thermodynamics
\[
-\frac{wn^2 H}{ikf} + (c - u_0)\Psi_z + u_0\Psi = 0
\]

\[
\Psi(y, z) = \phi(y, z)e^z
\]
\[
\phi(y, z) = Y(y)Z(z)
\]
\[
F_\ast = -\frac{u_{0zz} - u_{0z}}{u_0 - c} + \frac{\beta}{\epsilon(u_0 - c)} - \frac{k^2 + \ell^2}{\epsilon} - \frac{1}{4}
\]

\[
Y_{yy} + \ell^2 Y = 0
\]
\[
Z_{zz} + F_\ast Z = 0
\]

Assume \( c = 0 \) and \( u_0(z) \).
Boundary Conditions

at the surface, $z = 0$, $w = u \frac{db}{dx}$
assume a Gaussian mountain $\rightarrow b = e^{ikx} e^{-c^2(y-\frac{a}{4})^2}$

We require bounded solutions as our upper condition. $\rightarrow$ vertical group velocity is positive.

The vertical wave number is $m = \pm \sqrt{-F_{top}}$ and the group velocity is $c_g = \frac{d\omega}{dm}$.
Tropospheric Wave Reaction to Descending Deceleration

\[ u_i = 17 \text{ m/s}, \quad u_{ii} = 50 \text{ m/s}, \quad u_{iii} = 4 \text{ m/s} \]
\( u(i) = 7, \quad u(ii) = 32, \quad u(iii) = 20 \)

height (7 km = 1 scale height)
scale height of dissipation region

\[ u_i = 7 \text{ m/s}, \ u_{ii} = 60 \text{ m/s}, \ u_{iii} = 9 \text{ m/s} \]
Conclusions

Tropospheric Waves respond to Stratospheric changes in zonal wind speed (i.e. stratospheric changes in the index of refraction).